

Name: _____

Instructor: Jeffrey Diller

Math 230: Ordinary Differential Equations
Spring Semester 2001
Exam 1
Friday, March 2

This Examination contains 8 problems on 8 sheets of paper (including the front cover). Do all your work on the paper provided and show your computations. Do not use a calculator.

Scores

Question	Possible	Actual
1	10	
2	10	
3	10	
4	15	
5	15	
6	10	
7	10	
8	20	
Total	100	

GOOD LUCK

1. Solve the initial value problem

$$y' + 3y = e^t, \quad y(1) = 3.$$

(10 points)

2. Find the general solution of

$$\frac{d^4y}{dt^4} - 4y = 0.$$

(10 points)

3. Find the general solution of

$$2t^2y'' - 3ty' + 2y = 0$$

given that $t^{1/2}$ is a solution. (10 points)

4. Consider the initial value problem

$$y' = \sqrt{y} - t, \quad y(1) = 4$$

(a) Use Euler's method with a single step to approximate $y(1.01)$. (10 points)

(b) When I use Euler's Method with a step size of .01 to estimate $y(2)$ for this initial value problem, I get an approximate value of 4.593. When I drop the step size to .001, the approximation changes to 4.589. Based on this information, about what step size should I choose in order to estimate $y(2)$ to within 10^{-5} ? (5 points)

5. Consider the autonomous differential equation

$$y' = 2y^3(y^2 - 4).$$

(a) Identify each equilibrium solution of the equation and classify it as stable, unstable, or semistable. (10 points)

(b) On a single plot, sketch the graphs of the solutions y_1, y_2, y_3 satisfying $y_1(0) = 0$, $y_2(0) = -1$, and $y_3(0) = 4$. Make sure to label the graphs! (5 points)

6. (a) Show that the functions t, t^2, t^3 are linearly independent. (5 points)

(b) Suppose that $y''' + p_2(t)y'' + p_1(t)y' + p_0(t)y = 0$ satisfies the hypotheses of the existence and uniqueness theorem for linear ODE's for all $t \in \mathbf{R}$. Explain why t, t^2 , and t^3 cannot all be solutions of this ODE valid for all $t \in \mathbf{R}$. (5 points)

7. Bob, Peter, and Paul are roommates living in Bob's house. Bob owes Paul \$500 for all the CD's he borrowed and lost last year. In order to 'facilitate' Bob's repayment of this debt, Paul begins charging Bob interest at the rate of 10% per month. Bob decides that raising Peter's rent will be the most effective way to pay Paul back. Peter reacts by declaring his intention to vacate the premises in 6 months. Hence Bob needs your help to figure out how much he'll need to raise Peter's rent in order to pay Paul back before Peter leaves. Assuming that the debt accrues interest continuously and is paid off continuously, write down a differential equation and any relevant initial conditions that describe this situation. Let Bob find the solution himself. He's kind of annoying after all. (10 points)

8. Here is the direction field for a first order equation differential equation $y' = f(t, y)$. Decide whether each of the following statements is true or false, and **briefly** justify your answer. (5 points each)

(a) The differential equation is autonomous.

(b) There is a solution $y(t)$ that tends to -1 as $t \rightarrow \infty$ and to ∞ as $t \rightarrow -\infty$.

(c) Given $t_0 < 0$ and $y_0 < 0$, the solution satisfying $y(t_0) = y_0$ satisfies $y(t) < 0$ for all $t > t_0$.

(d) $y = t$ is a solution of the equation.