Name: ________________________________ **Instructor:** Jeffrey Diller

Math 230: Ordinary Differential Equations
Spring Semester 2001
Exam 2
Friday, April 20

This Examination contains 5 problems on 7 sheets of paper (including the front cover). Do all your work on the paper provided and show your computations. Do not use a calculator.

Scores

Question	Possible	Actual
1	25	
2	20	
3	30	
4	15	
5	10	
Total	100	

GOOD LUCK

1. Consider the following inhomogeneous differential equation

$$y''' + 2y'' + y' = \cos t - 2e^{-t}.$$

(a) What form will the general solution of this equation have? Circle the arbitrary constants that must be determined, but do **NOT** try to actually solve for them. (15 points)

(b) Rewrite this equation as a first order system of equations, and express the system in matrix form. (10 points)

2. Consider the system

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 4 & -1 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

(a) Find the general solution. (15 points)

(b) What further equations must you solve to find the particular solution satisfying

$$\begin{pmatrix} x_1(\frac{\pi}{2}) \\ x_2(\frac{\pi}{2}) \end{pmatrix} = \begin{pmatrix} \pi \\ e^2 \end{pmatrix}?$$

You need not actually solve the equations, but I do expect you to know the values of trig functions at basic angles. (5 points)

3. Suppose that $\mathbf{x}' = A\mathbf{x}$ is a 2×2 system of ODE's, and that A is a constant matrix. Suppose that

$$\mathbf{x}^1(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t} \qquad \text{and} \qquad \mathbf{x}^2(t) = \begin{pmatrix} -2 \\ 3 \end{pmatrix} e^{3t}$$

are solutions to this system.

(a) What is e^{At}? You don't need to simplify your answer to this one. (5 points)

(b) What sort of critical point is $(0, 0)$ for this system? Is it stable or unstable? (5 points)

(c) On the graph below, plot solution curves for solutions $\mathbf{x}(t)$ satisfying $\mathbf{x}(0) = (1/2, 1)$ and $\mathbf{x}(0) = (0, -1)$. Draw arrows on your solution curves to indicate the direction for increasing t. (10 points)

(d) Suppose that we change the system to

$$\mathbf{x}' = A\mathbf{x} + \begin{pmatrix} t^2 \\ t^3 \end{pmatrix}$$

where A is the same matrix as before. Write down the solution to this new system. *Do* **not** *simplify your answer—i.e. don't perform any matrix multiplication, inversion, integration, differentiation, etc. (10 points)*

4. Each of the following three pictures represents a direction field or phase portrait for a system of ODE's of the form

$$\mathbf{x}' = A\mathbf{x},$$

where $\mathbf{x}(t) = (x_1(t), x_2(t))$ is a vector-valued function and A is 2×2 constant matrix. For each picture, say what can be said about the eigenvalues and eigenvectors of the matrix A. (5 points each)

(a)

(b)

(c)

5. Suppose that A is a 2×2 constant matrix and that $B = 2A$. Compare solutions of the system

$$\mathbf{x}' = A\mathbf{x}$$

with those of

$$\mathbf{x}' = B\mathbf{x}.$$

For instance, do the direction fields and phase portraits of the two systems have anything in common? How about the actual formulas for the general solutions? Try to justify your responses. It might also help to consider a particular example. (10 points)