MATH 261 – LINEAR ALGEBRA

FALL 1999 (EXAM 1)

(1) Consider the subsets of $\mathbb{R}^3$ given by

$$A = \{(1,1,0)\ ,(0,1,1)\}, \quad B = \{(1,2,1),(1,0,-1),(3,4,1)\}.$$

Show that the subspaces generated by A and B are the same.

(2) Consider the subspace W of $\mathbb{R}^4$ generated by the vectors

$$(1,0,0,3),\ (-1,-2,0,1),\ (1,2,3,-2).$$

Does $(2,3,-1,5) \in W$?

(3) Is the set $\{(1,0,1),(0,1,1),(1,1,2)\} \subset \mathbb{R}^3$ linearly independent?

(4) Let V be an n-dimensional vector space and $A \subset V$ a set with n elements. Suppose that $\mathcal{L}\{A\} = V$. Must A be a basis for V? Either prove that the answer is *yes* or give a counterexample.

(5) Let V be a vector space and consider the *diagonal* set $\Delta \subset V \times V \times ...V$ (p times) given by

$$\Delta = \{(v,v,...,v)\ |v \in V\}.$$

(i) Show that Δ is a subspace of $V \times V \times ...V$.
(ii) If $\dim V = n$, what is $\dim \Delta$?

(6) Define what it means for a vector space V to be the direct sum of non-trivial subspaces W_1 and W_2. For each k, $0 < k < n^2$, exhibit the space $M^{n\times n}$ of $n \times n$ matrices as a direct sum where one of the subspaces has dimension k.

(7) Let W_1 and W_2 be subspaces of the vector space V such that $W_1 \cup W_2$ is also a subspace. Show that one of the subspaces is contained in the other.

(8) Give infinitely many distinct examples of infinite - dimensional vector spaces.