MATH 261 – LINEAR ALGEBRA

FALL 1999 (EXAM 3)

(1) Give an example of an invertible linear transformation from $\mathbb{R}^3$ into itself, other than the identity, which has a two-dimensional invariant subspace.

(2) a) Let $T : \mathbb{R}^m \to \mathbb{R}^n$ be a linear map, where $m < n$. Show that T is not surjective.
b) Restate a) as a theorem about non-homogeneous linear systems of equations.

(3) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by $T(x, y) = (2x + y, x - 3y)$. Let β be the canonical basis of $\mathbb{R}^2$ and let $\beta' = \{(1, 1), (1, 2)\}$. Use theorem 2.23 to find $[T]_{\beta'}$.

(4) a) Consider the linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ which consists in rotating counterclockwise around the origin by $\frac{\pi}{2}$. Write down the matrix of T relative to the canonical basis.
b) What are the invariant subspaces of T?

(5) Let V be a finite dimensional vector space and W a proper subspace of V. Let a be in the complement of W in V. Show that there is a linear functional $v^* \in V^*$ such that $v^*(a) = 1$ and $v^*(w) = 0$ for all $w \in W$. Hint: complete a basis for $\mathcal{L}\{W, a\}$ to a basis of V and then consider its dual.

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