MATH 261 – LINEAR ALGEBRA

FALL 1999 (FINAL EXAM)

(1) What linear relation(s) must (w_1, w_2, w_3, w_4) satisfy in order for it to be in the range of $T : \mathbb{R}^3 \to \mathbb{R}^4$, $T(x_1, x_2, x_3) = (3x_1 - x_2 - 4x_3, 2x_2 + 3x_3, 3x_1 - 2x_2 + x_3, 4x_1 + x_2 - x_3)$?

(2) Show that $\mathbb{R}^2$ has a basis β consisting of eigenvectors of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x + 2y, 2x + y)$. Write down the matrix $[T]_\beta$.

(3) Show that *similar* linear maps $S, T : V \to V$ have the same nullity.

(4) Is there an *injective* linear transformation from the space of all $n \times n$ symmetric real matrices into $\mathbb{R}^{2n}$? Justify your answer.

(5) Prove that two finite-dimensional vector spaces (over the same field) are isomorphic if and only if they have the same dimension.