Math 261 – Linear Algebra
Fall 2000 (Exam 1)

1. *(15 points)* Let $W_1 = \{(a,b) \in \mathbb{R}^2 \mid a + 2b = 0\}$ and $W_2 = \{(a,b) \in \mathbb{R}^2 \mid a + 2b = 1\}$. Decide which of W_1 and W_2 are subspaces of $\mathbb{R}^2$. Prove your answer.

2. *(15 points)* Consider the two subsets A and B of $\mathbb{R}^3$ given by

$$A = \{(1,1,0)\ ,(0,1,1)\}, \quad B = \{(1,2,1),(1,0,-1),(3,4,1)\}.$$

Show that the subspaces of $\mathbb{R}^3$ spanned by A and B are the same.

3. *(15 points)* Consider the subspace W of $\mathbb{R}^4$ spanned by the vectors

$$(1,0,0,3),\ (-1,-2,0,1),\ (1,2,3,-2).$$

Is $(2,3,-1,5) \in W$?

4. *(15 points)* Is the set $\{(1,-1,1),(1,1,1),(1,0,1)\} \subseteq \mathbb{R}^3$ linearly independent? If not, find a basis for the subspace of $\mathbb{R}^3$ spanned by this set.

5. *(15 points)* Let F be a field. Consider the subset

$$W = \{(\lambda_1, \ldots, \lambda_n) \in F^n \mid \lambda_1 + \ldots + \lambda_n = 0\}$$

of F^n, which you may assume is a subspace of F^n. Find a basis for W and determine the dimension of W as a F-vector space.

6. *(25 points)* **(a)** Carefully state the definition of the span of a subset S of a F-vector space V, and the definition of linear independence of S.

(b) Let V be a vector space and $\{v_1, \ldots, v_n\}$ be a set of n vectors in V. Fix an integer i with $1 \leq i \leq n-1$. Prove that $\{v_1, \ldots, v_n\}$ is linearly independent if and only if the following three conditions all hold: $A = \{v_1, \ldots, v_i\}$ is linearly independent, $B = \{v_{i+1}, \ldots, v_n\}$ is linearly independent and $\operatorname{span}(A) \cap \operatorname{span}(B) = \{0\}$.