Mathematics 262
Homework 3 solutions

Assignment: Section 5.4, problems 1, 2, 14

§5.4, #1: You just use the formula. The inverses of the two given matrices are:

$$\frac{1}{72}\begin{bmatrix} -3 & 5 & 9 \\ 18 & -6 & 18 \\ 6 & 14 & -18 \end{bmatrix}, \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.$$

There's no real excuse for getting these wrong, since you can find the inverse other ways, and since it's easy to check your work (just multiply the original matrix by the supposed inverse). Of course, I probably screwed up and gave you the wrong answers, but I'm not getting graded on this...

The second matrix in the problem has a geometric interpretation: it corresponds to rotating $\mathbf{R}^3$ around the y-axis by the angle $-\theta$. The inverse matrix should precisely undo this, so it's not surprising that the inverse matrix rotates $\mathbf{R}^3$ around the y-axis by the angle θ.

§5.4, #2: Again, you just use the formula. For part (a), the system of equations is

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -6 & -1 \\ 3 & 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \\ 0 \end{pmatrix}.$$

The determinant of the coefficient matrix is 11, so according to Cramer's rule,

$$x = \frac{\det \begin{pmatrix} 11 & 1 & 1 \\ 0 & -6 & -1 \\ 0 & 4 & 2 \end{pmatrix}}{11} = -8.$$

Similarly, $y = -7$ and $z = 26$. As with the previous problem, since you can check your work by plugging in the numbers, you really should have the right answer.

For part (b), the system is

$$\begin{pmatrix} 3 & -2 & 0 \\ 0 & 3 & -2 \\ -2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ -1 \end{pmatrix}$$

Since the determinant of the coefficient matrix is 19, then

$$x = \frac{\det \begin{pmatrix} 7 & -2 & 0 \\ 6 & 3 & -2 \\ -1 & 0 & 3 \end{pmatrix}}{19} = 5.$$

Similarly, $y = 4$ and $z = 3$.