Final Exam

Due at noon, Thursday, May 7 in 219 CCMB

General instructions: This is a closed book, closed notes exam. You may not use calculators, computers, etc. You may not discuss the exam with anyone except me. This exam is being conducted under the honor code. You may spend up to two hours working on the exam, preferably in one or two sittings.

If you need to reach me: E-mail: `John.H.Palmieri.2@nd.edu`. Office: 219 CCMB. Phone: 631-5352. I expect to be in my office most of this week (but you might want to call before you come by, just to be sure that I'm there).

Definitions

(10) 1. Let $T : V \to V$ be a linear operator. Define what it means for a subspace W of V to be *invariant*. Give me an example of an invariant subspace (so tell me what V, T, and W are, and briefly explain why W is invariant).

(7) 2. Define *inner product*.

(8) 3. Describe the *Gram-Schmidt orthogonalization* procedure.

Short answer

4. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

(5) (a) Compute $\det A$.

(5) (b) Compute the "classical adjoint" of A, $\operatorname{adj}(A)$. (This is the adjoint that has to do with computing inverses of matrices, *not* the adjoint involving inner products.)

(5) (c) Give the general formula for A^{-1} in terms of $\operatorname{adj}(A)$. Compute A^{-1} in this particular case.

(10) 5. Let $B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 0 \\ 3 & 6 & 1 & 2 & 3 \\ 4 & 8 & 2 & 5 & 7 \\ 5 & 0 & 3 & 7 & 0 \end{bmatrix}$. Without computing the characteristic polynomial of B (or doing anything similarly time-consuming), what can you tell me about its eigenvalues and eigenvectors? Is B diagonalizable (over $\mathbf{R}$)?

(10) 6. Let $C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. Is C similar to a diagonal matrix (over $\mathbf{R}$)?

7. Let $D = \begin{bmatrix} 6+i & -2-2i \\ -2-2i & 9+4i \end{bmatrix}$.

(7) (a) Is D normal (with respect to the standard inner product on $\mathbf{C}^2$)?

(8) (b) Compute the eigenvalues and eigenvectors of D.

Theory

(5) 8. (a) State the Cayley-Hamilton theorem.

(10) (b) Outline its proof. (Outline it, don't give me all of the details.)

(15) 9. Outline the proof of the theorem that says $\det(AB) = \det(A)\det(B)$.

10. Let A be an $n \times n$ matrix.

(5) (a) Can you tell whether A is invertible by looking at its eigenvalues?

(15) (b) How can I tell if A is invertible? (We have lots of different tools available for this; list as many as you can, tell me advantages and/or disadvantages of each, give examples, etc.)

(25) 11. How can I tell if an $n \times n$ matrix is similar to a diagonal matrix? (We discussed at least two theorems related to this. State them and any related results, outline their proofs, give examples, etc.)