

Math 262
Spring 1999
Test 1

1. Write an essay about $F[X]$, the ring of polynomials with coefficients in the field F .
2. Prove that a real 2×2 symmetric matrix is diagonalizable.
3. Let N be a 2×2 complex matrix, $N^2 = 0$. Prove that either $N = 0$ or N is similar to $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

Hint: If $N \neq 0$ then $\dim \ker N = 1$ (why?). Look at the effect of N on a basis $\{\alpha, \beta\}$ with $\beta \in \ker N$.

4. Any 2×2 complex matrix A is either diagonalizable or A is similar to a matrix of the form $\begin{bmatrix} a & 0 \\ 1 & a \end{bmatrix}$

Hint: In case the eigenvalues of A are equal to c , show that the eigenvalues of $N = A - cI$ are all zero. Conclude that N is nilpotent and use problem 3.