ath 262 – Linear Algebra

Spring 2000 (Exam 1)

1) Define $\det A$ and state the basic properties of the function $\det : M_{n\times n}(F) \to F$.

2) Let $N \in M_{n\times n}(F)$ be similar to an upper triangular matrix whose main diagonal consists entirely of zeros. Show that $I + N$ is invertible.

3) A matrix $A \in M_{n\times n}(F)$ is said to have a *square root* if there exists a matrix $X \in M_{n\times n}(F)$ such that $X^2 = A$.

a) Show that if $F = \mathbb{R}$ and n is odd then $A = -I$ does not have a square root.

b) Find a square root of $A = -I$ when $F = \mathbb{R}$ and $n = 2$.

4) Consider the linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y, z) = (x, x + 2y + 3z, x + 2z)$.

a) Write down the characteristic polynomial of T.

b) Find the eigenvalues of T.

c) Describe the eigenspace of T correponding to the *double* eigenvalue.

d) Is T diagonalizable? Explain.