ath 262 – Linear Algebra

Spring 2000 (Exam 2)

1) Let $T : V \to V$ be a linear operator, where V is a finite-dimensional complex-vector space. Show that there exists a basis of V with respect to which the matrix of T is upper triangular.

2) Show by means of an example that the conclusion in problem 1 may fail if V is a real vector space.

3) Consider the operator $T : \mathbb{C}^4 \to \mathbb{C}^4$ given by

$$T(z_1, z_2, z_3, z_4) = (z_1 + iz_2 + z_3 + z_4, z_2, z_2 + 3z_4, z_2 + 4z_4).$$

Is T diagonalizable? What is the minimal polynomial of T?

4) Linear operators on a complex infinite – dimensional vector space need not have eigenvalues. Check that the operator below is a suitable example: V = space of all continuous functions $f : [0, 1] \to \mathbb{C}$, $T : V \to V$, $Tf = gf$, where g is a fixed function in V with the property that g is non-constant on every open subinterval of $[0, 1]$.