ath 262 – Linear Algebra

Spring 2000 (Exam 3)

1) Write an essay about the Jordan canonical form for operators on finite - dimensional complex vector spaces. Your discussion must include the primary decomposition theorem, the decomposition of an operator in terms of diagonalizable and nilpotent operators, etc... You must explain clearly how these elements fit together to yield the Jordan form. Give at least two examples to illustrate the various possibilities.

2) Let V be a finite - dimensional inner product space, V^* its dual space. Prove that there is a *natural* isomorphism $\Lambda : V \to V^*$ (give full details).

3) Denote by $O(3)$ the set of all orthogonal linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $\det T = 1$. Show that $O(3)$ is a group under the usual notion of product of operators. For a fixed $p \in \mathbb{R}^3$, what is the subset of $\mathbb{R}^3$ defined by $\{Tp \mid T \in O(3)\}$?

4) Apply the Gram - Schmidt orthogonalization process to your favorite non- orthogonal basis of $\mathbb{R}^4$.

5) Using the spectral theorem, explore the possibility of defining $f(A)$, where $f : \mathbb{R} \to \mathbb{R}$ is any function and A is any symmetric $n \times n$ real matrix.