ath 262 – Linear Algebra

Spring 2000 (Final Exam)

1) Prove the *parallelogram law* on an inner product space V; that is, show that for all vectors x and y in V

$$||x+y||^2 + ||x-y||^2 = 2||x||^2 + 2||y||^2.$$

What does this equation say about parallelograms in $\mathbb{R}^2$?

2) Consider the space $M_{n\times n}(\mathbb{R})$ of all $n \times n$ real matrices endowed with the inner product $\langle A, B\rangle =$ trace of B^*A. Consider the subset W of $M_{n\times n}(\mathbb{R})$ consisting of those matrices of zero trace. Show that W is a subspace of dimension $n^2 - 1$ and find its orthogonal complement.

3) Give examples of:

a) a 3×3 nilpotent matrix of rank 2.

b) A 9×9 matrix in Jordan form with three Jordan blocks and a double eigenvalue.

4) Let N be a nilpotent $n \times n$ matrix. Show that the matrix $I - N$ is invertible (you may either use determinants or look at the power series expansion about $x = 0$ of the function $(1-x)^{-1}$ for inspiration).

5) let V be a 14-dimensional vector space, W_1, W_2 subspaces of dimensions 8 and 10, repectively. What are the possibilities for the dimension of $W_1 \cap W_2$?

6) a) Give the definition of the minimal polynomial associated to an operator. b) Formulate and prove a criterion for an operator on a finite – dimensional vector space to be triangulable.