6 Honors Calculus IV   Spring 1997

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xt: Tom M. Apostol, Calculus, Vol. 2, Second edition, John Wiley & Sons, 19

terial covered:

Line integrals.

Paths, simple paths, Jordan curves. Definition of the line integral of a
ctor
field along a path. (Sections 10.2 and 10,3)

Properties of line integrals: Linearity with respect to the integrand and
additivity with respect to the path. Change of parameter. (Section 10.4)
Interpretation of the line integral as work. (Section 10.6)

Arc length and the line integral of a scalar field with respect to arc
ngth.

Applications: Mass, moments of inertia, coordinates of the center of mass
(Sections 10.7 and 10.8)

Independence of the path. Second fundamental theorem of calculus for line
integrals.

Conservative fields, potential. (Sections 10.11 and 10.12)

The first fundamental theorem of calculus for line integrals.(Sections
10.13-10.14)

Vector fields and gradients (Sections 10.15, 10.16 and 10.17)

Exact differentials. Integrated factor (Section 10.19).

Finding the potential (Section 10.21)

Multiple integrals

Definition of a double integral over an interval. Integrable functions.
Properties of the double integral: Linearity with respect to the integrand
d
additivity with respect to the domain of integration. (Sections 11.1-11.5)

Representation of a double integral as an iterated integral. (Section 11.6

Integrability of a continuous function. (Section 11.10)

Bounded sets of content zero. Integrability of functions with
scontinuities.

(Section 11.11).

Double integrals over general regions and their representation as iterated integrals. (Section 11.2).

Applications. Volumes, mass, moments of inertia, coordinates of the center
:
mass. (Sections 11.13 and 11.16)

Green's theorem and its applications. (Sections 11.20 and 11.21)

Generalization of Green's theorem to multiply connected regions (Section
..24).

Change of variables in the double integrals. (Section 11.26)

Change to polar coordinates and linear transformations. (Section 11.27)

Proof of the theorem about change of variables. (Sections 11.29 and 11.30)

Generalization to higher dimensions (Section 11.31)

Change of variables in an n-fold integral. Cylindrical and spherical
oordinates
in 3-dimensional apace.(Sections 11.22 and 11.33).

Surface integrals.

Parametric representation of surfaces. Geographic coordinates on the spher

Parametrization via stereographic projection. Surfaces of revolution.
ection
12.1)

The fundamental vector product . The unit normal vector. Orientation of a surface.

The Moebius band.(Sections 12.3 and 12.4).

Area of a parametric surface. Independence of the parametrization. Area of surface of revolution. The theorem of Pappus. (Section 12.5)

Surface integrals. Independence of the parametrization. (Sections 12.7 and
.8)

Alternate notation for surface integrals. (Section 12.9)

Gradient, curl, divergence and relations between them. (Sections 12.10 and
.14)

The theorem of Stokes. (Sections 12.11 and 12.18)

The divergence theorem and its applications (Sections 12.19 and 12.20)

An overview of differential forms.

A formal definition a differential form of degree k in R^n). Addition of differential forms of the same degree.

Exterior product of differential forms. The pullback of a differential form
a
mapping $F: R^m \to R^n$.

The integral of a differential form of order k over an oriented k-
mensional
surface in R^n. The differential of a form. Closed and exact forms.

The general Stokes theorem.

Illustration of the dualism between vector fields and differential forms in dimension 3.