UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

MATH 318 Examination 1 October 16, 2002

Name: ____________________________

The Honor Code is in effect
The format is "open−book" and "open−notes"
The time limit is 50 minutes

Prob. 1	/ 10
Prob. 2	/ 10
Prob. 3	/ 10
Total	/ 30

1. (a) [6 points] A function $y(x)$ has given values $y(a)$, $y(a + h)$, and $y(a + 3h)$.

Develop the highest−order finite−difference formula for the first derivative $y'(a)$ that can be obtained in terms of the three given values. Please show the major steps of your work. [You will need the order of the error for part (b)].

(b) [4 points] One can use the formula of part (a) to approximate $y'(2.00)$ if one knows y at $(x = 2.00)$, at $(x = 2.03)$, and **either** at $(x = c)$ **or** at $(x = d)$. Assume that c is less than d, and let the errors in approximating $y'(2.00)$ be $E(c)$ when the y value at $(x = c)$ is used and $E(d)$ when the y value at $(x = d)$ is used. Based on the order of the error in the difference approximation, one expects $E(d)/E(c)$ to be very nearly equal to an integer k.

Give the values of c, d, and k, and explain briefly how you obtained them.

2. Consider

$$\mathbf{A} = \mathbf{LU} = \begin{bmatrix} -1 & 2 & 0 \\ 2 & -1 & -1 \\ -2 & 0 & k+2 \end{bmatrix}; \quad \mathrm{Det}(\mathbf{A}) = d$$

where **L** and **U** are the Doolittle lower and upper triangular factors, respectively.

(a) [3 points] Determine k such that d is equal to k. Please show your work.

(b) [5 points] For all k such that $\mathbf{B} = \mathbf{A}^{-1}$ exists, determine element b_{23} of **B** and the ratio $\mathrm{Det}(\mathbf{A}) / \mathrm{Det}(\mathbf{B})$ in terms of d, and give element l_{31} of **L** as a pure number. Please show or explain major steps of your work.

(c) [2 points] Columns 1 and 2 of **A** are swapped to produce a tridiagonal matrix **T**, State **clearly and unambiguously** which one of the following is true.

1. $\mathrm{Det}(\mathbf{T}) = d$. **2.** $\mathrm{Det}(\mathbf{T}) = -d$. **3.** $\mathrm{Det}(\mathbf{T})$ is neither d nor $-d$.

3. [10 points] A function $y(x)$ satisfies the boundary−value problem

Differential equation: $xy'' + 2y' = 2; \qquad 1 \le x \le 4$

Boundary conditions: $y(1) = 17; \qquad y'(4) = 0$

in which the primes denote differentiation with respect to x.

Define: $y_i = y(x_i); \qquad x_i = i + 1, \quad i = 0, 1, 2, 3$

Use 2nd−order, central−difference approximations for the derivatives in the differential equation to recast the problem as a system of linear algebraic equations

$$\mathbf{Ay} = \mathbf{f}; \qquad \mathbf{y} = [\, y_1 \quad y_2 \quad y_3 \,]^{\mathrm{T}}$$

Please show your basic procedure, and **give complete representations** of **A** and **f**.

Hint: Combine the right boundary condition with the 2nd−order, central−difference formula for the first derivative to eliminate any y value that does not actually exist.