UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

MATH 318 Examination 2 November 25, 2002

Name: ____________________________

The Honor Code is in effect
The format is "open−book" and "open−notes"
The time limit is 50 minutes

Prob. 1	/ 10
Prob. 2	/ 10
Prob. 3	/ 10
Total	/ 30

1. A consistently−ordered system of linear algebraic equations with a two−cyclic co−efficient matrix is to be solved by successive over−relaxation (SOR). The spectral radius ρ of the SOR iteration matrix is 0.98334 when the acceleration parameter ω is equal to 1.6. Assume that this value of ρ is accurate.

(a) [1 point] Explain **briefly and clearly** why we can conclude that the ω value 1.6 is less than the optimum value.

(b) [2 points] Let $E(k)$ be an error vector norm at iteration k, and assume an ideal convergence behavior such that $E(k) / E(k-1)$ is equal to ρ. Find the smallest integer κ for which $E(\kappa) / E(0)$ is less than or equal to 0.0000001 when $\omega = 1.6$ is the acceleration parameter. **Please show your work**.

(c) [7 points] Determine the optimum acceleration parameter Ω (after first finding the Gauss−Seidel spectral radius). **Please show your work, use a high level of precision for intermediate calculations, and give your answer to at least four decimal places**.

2. (a) [6 points] Consider the integral $I = \int_1^9 f(x)\,dx$

An estimate of I by Simpson's rule with two equal intervals is 668, and another estimate of I by the trapezoidal rule with four equal intervals is 593. Also, two values of the integrand are $f(1) = 1$ and $f(9) = 46$.

Romberg estimates of I are denoted by $I(r, c)$, where r indicates the use of 2^r equal intervals and c is the level of correction to the trapezoidal rule. Give the numerical values of the Romberg estimates in the table below. **Grades for this part are given strictly for results in the table!**

$I(0, 0) =$		
$I(1, 0) =$	$I(1, 1) =$	
$I(2, 0) =$	$I(2, 1) =$	$I(2, 2) =$

(b) [4 points] Estimate $\int_1^5 [1/(1 + \ln x)]\,dx$ by a two−point Gauss quadrature.

Please show adequate work and give your result to at least 4 significant digits.

3. (a) [2 points] Consider **[1]** $v = 0.5(\theta + v) - 0.125\sin(2v)$

[2] $v = \theta - 0.25\sin(2v)$

Equation [1] has a single solution v for a given θ value (with both v and θ in radians). Is Eq. [2] equivalent to Eq. [1]? In other words, is the v solution of Eq. [1] also the solution of Eq. [2]? **Answer unambiguously!**

(b) [2 points] A proposed fixed−point iteration scheme for solving Eq. [1] above is

$$v[k] = 0.5(\theta + v[k-1]) - 0.125\sin(2v[k-1])$$

in which $v[k]$ is the k−th estimate of v. Which **one** of the following is true?

1) The scheme will not converge for all starting values $v[0]$.

2) The scheme converges for all $v[0]$ **and** errors in successive v estimates have opposite signs.

3) The scheme converges for all $v[0]$ **and** errors in successive v estimates have the same signs.

(c) [6 points] Obtain the positive solution x of: $x - \sin x = 1$

Use any legitimate method, show basic steps, and report the result so that the left and right sides of the given equation differ by no more than 0.00001 in magnitude.