= 4 Math 323 Test 3 April 6,1994

The random variables X_1 and X_2 have joint density function

$$f(x_1,x_2) = \begin{cases} 24x_2 & \text{if } 0 \le x_2 \le x_1 \le 1 \text{ and } x_1 + x_2 \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the marginal density function $f_2(x_2)$.

$24(x_2 - 2{x_2}^2)$ $24(x_2 - {x_2}^2)$ $24(x_2 + 2{x_2}^2)$ $24(x_2 + {x_2}^2)$ $24(\frac{1}{2} - x_2 + {x_2}^2)$ 1:bcaed 2:acebd 3:dbcea 4:cadbe

For X_1 and X_2 of problem 1, find the marginal density function $f_1(x_1)$. $12(\frac{1}{2} - x_1 + {x_1}^2 - |x_1 - \frac{1}{2}|)$ $24(x_1 - 2{x_1}^2)$ $24(-x_1 + {x_1}^2)$ $24(\frac{1}{2} - x_1 - {x_1}^2)$ $24(\frac{1}{2} + x_1 + {x_1}^2)$ 1:cbdae 2:bdaec 3:daecb 4:aecbd

Let X_1 and X_2 have a uniform joint probability function given by

$$f(x_1,x_2) = \begin{cases} \frac{1}{2} & \text{if } 0 \le x_1 \le 2 \text{ and } 0 \le x_2 \le 1 \\ \\ 0 & \text{elsewhere.} \end{cases}$$

Determine the probability $P(X_1 + X_2 \le 1)$. $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{3}{8}$ 1:abcde 2:acebd 3:cbaed 4:cadbe

Two telephone calls come independently into a switchboard, and at random times in a fixed 1-hour period. What is the probability that they arrive within 10 minutes of each other? $\frac{11}{36}$ $\frac{21}{36}$ $\frac{13}{36}$ $\frac{15}{36}$ $\frac{25}{36}$

1:acbed 2:cbeda 3:bedac 4:adbec

The standard normal variable Z has $e^{\frac{t^2}{2}}$ as its moment-generating function. Using this information, compute $Var(Z^2)$.

2 3 1 4 5 1:baced 2:acedb 3:cedba 4:edbac

Let X be normally distributed with $\mu = 600$ and $\sigma = 30$. Compute $P(X > 660)$. .0228 .4772 .9772 .5228 .0013 1:cbade 2:badec 3:adecb 4:decba

Professor Stern just gave a mathematics test to his students. The scores are normally distributed with a mean of 75 and a standard deviation of 16. What should the lowest passing grade be so that 10% of the class fails the test? 55 60 52 58 62 1:bdaec 2:daecb 3:aecbd 4:ecbda

Certain steel beams have yield strengths (measured in thousands of pounds per square inch) which follow a Weibull distribution with $\gamma = 2$ and $\theta = 4{,}000$. A construction project uses two such beams and calls for a yield strength in excess of 70,000 psi. What is the probability that both beams meet the specifications for the project? .086 .590 .228 .387 .291 1:edcba 2:daecb 3:aecbd 4:ecbda

A random variable X has a beta distribution with $\alpha = 3$ and $\beta = 1$. Find $P(X > \frac{1}{2})$. $\frac{7}{8}$ $\frac{15}{16}$ $\frac{5}{8}$ $\frac{13}{16}$ $\frac{7}{16}$ 1:abcde 2:bcdea 3:cdeab 4:deabc

Let X be exponential with density function $\frac{1}{2}e^{\frac{-x}{2}}$ for $x > 0$. Find the moment-gemerating function of X. $(1-2t)^{-1}$ $(2-t)^{-1}$ $(1-t)^{-2}$ $(2t-1)^{-1}$ $(t-2)^{-2}$ 1:bacde 2:acdeb 3:cdeba 4:debac

Illustrate the usefulness of the integration formula $\int_0^\infty x^n e^{\frac{-x}{\theta}}\,dx = \Gamma(n+1)\,\theta^{n+1}$ by computing $E[X^3]$ where X is a gamma random variable with $\alpha = 2$ and $\beta = 4$.

1536 $\frac{1}{14}$ 24576 7680 $\frac{1}{64}$ 1:baecd 2:aecdb 3:ecdba 4:cdbae

The joint probability function for the discrete random variables X_1 and X_2 is 0 everywhere except for the four points (1,1), (1,2), (2,1), and (2,2). At (2,2), p(2,2) = $\frac{5}{8}$ and p(1,1) = p(1,2)= p(2,1) = $\frac{1}{8}$. Find the expected value $E[X_1X_2]$ of the product X_1X_2.

$\frac{25}{8}$ $\frac{9}{8}$ $\frac{28}{8}$ $\frac{17}{8}$ $\frac{13}{8}$ 1:ecbda 2:bdaec 3:dabce 4:dabce

Let the joint density function of X_1 and X_2 be given by $f(x_1,x_2) = 2x_1$for points (x_1,x_2) in the unit square and $f(x_1,x_2) = 0$ elsewhere. Find the probability

$$P(X_2 \le \frac{1}{8} \mid X_1 \le \frac{1}{4}).$$

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{3}{8}$ 0 1:baecd 2:cdbae 3:dbaec 4:ecdba

For X_1 and X_2 of problem 13, find $P(X_2 \le \frac{1}{8} \mid X_1 = \frac{1}{4})$. $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{16}$ $\frac{3}{16}$ 1:ecbad 2:decba 3:ecbad 4:cbade

Are the random variables X_1 and X_2 of problems 13 and 14 independent or not independent?

independent not independent don't choose this don't choose this don't choose this 1:cbeda 2:edacb 3:dacbe 4:cbeda

The table below gives the joint probability function for the discrete random variables X_1 and X_2. Use it to find $P(X_1 \le 1 \mid X_2 = 1)$.

$\frac{27}{45}$ $\frac{9}{28}$ $\frac{2}{7}$ $\frac{3}{14}$ $\frac{10}{21}$ 1:cdeab 2:cdeab 3:deabc 4:deabc