MATH 323 TEST 3 April 28, 1995

1. Let $f(x) = 3x^2 \quad 0 \le x \le 1$

   0 elsewhere

   Then $P(.3 \le X \le .8) =$

a. 0.485 b. 0.512 c. 0.027 d. 0.064 e. 0.217

2. Heavy rains occur twice every three years on the average in a certain country. The variance in time (in years) between heavy rains in that country is

a. 2.25 b. 1.5 c. 0.73 d. 4.5 e. 0.44

3. A random variable X has exponential distribution with mean 4. The value of $E(X^3)$ is

a. 384 b. 192 c. 251 d. 64 e. 128

4. Let X be a random variable with gamma distribution with $\alpha = 700$ and $\beta = 50$. A lower bound for the probability $P(X \leq 40000)$ according to Tchebycheff's inequality is (hint: $E(X) = 35000$)

a. 0.93 b. 0.87 c. 0.07 d. 0.78 e. 0.66

5. Let X be normal with mean 500 and standard deviation 60. Then $P(500 \leq X \leq 602)$ is

a. 0.455 b. 0.066 c. 0.044 d. 0.233 e. 0.031

6. Let $f(x) = kx^3(1-x)^5$ for $0 \leq x \leq 1$

0 elsewhere

What value of k will make this a probability density?

a. 504 b. 168 c. 105 d. 464 e. 398

7. Let X have Beta distribution with $\alpha = 2$, $\beta = 3$. Then $E(X^2)$ is

a. 0.200 b. 0.181 c. 1.81 d. 2.03 e. 0.562

8. The number of cars arriving at a toll booth has a Poisson distribution with a rate of 20 per hour. Suppose that a car has just arrived and X is the time until the next car comes. The probability density function for X on the interval $(0 < t < \infty)$ is f(t) =

a. $0.05e^{-0.05t}$ b. $20\, e^{-20t}$ c. $\frac{20^t}{t!}\, e^{-20}$ d. $e^{-t/20}$

e. $\frac{1}{t-20}$

9. The amount of a certain drug made by a firm in one day has an exponential distribution with a mean of 3 lb. The probability that the firm produces more than 6 lb in one day is

a. 0.1353 b. 0.7413 c. 0.2217 d. 0.0598 e. 0.0761

10. Let X_1 and X_2 be gamma distributed with parameters $\alpha_1 = 3$, $\beta_1 = 3$, $\alpha_2 = 4$, $\beta_2 = 3$. Then $Y = X_1 + X_2$ will be gamma distributed with parameters

a. $\alpha = 7$, $\beta = 3$ b. $\alpha = 7$, $\beta = 6$ c. $\alpha = 1$, $\beta = 9$

d. $\alpha = 12$, $\beta = 6$ e. $\alpha = 12$, $\beta = 9$

11. The height at the shoulders of a adult bull elephant has normal distribution with mean 3.3 meters and standard deviation 0.2 meters. The probability that an elephant will be found whose height at shoulders is larger than 4 meters is

a. 0.0002 b. 0.0012 c. 0.0110 d. 0.0022 e. 0.0004

12. Let X be the time (in hours) necessary for a certain type of epoxy to cure, and suppose that X has Weibull distribution with $\gamma = 2$ and $\theta = 1.5$. The probability that it will take less than $\sqrt{3}$ hours to cure is

a. 0.865 b. 0.912 c. 0.711 d. 0.157 e. 0.665

13. A final exam consisting of 150 questions is given, and the scores were found to be approximately normally distributed with mean 113 and standard deviation 28. What is the lowest grade for passing if 90% of the students will pass?

a. 78 b. 61 c. 84 d. 92 e. 53

14. Let X_1 and X_2 be jointly uniform on the region defined by $0 \le x_1 \le 1$, $0 \le x_2 \le 2$. The joint probability density function on this region is $f(x_1, x_2) =$

a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. 1 d. 2 e. $\frac{3}{2}$

15. Let $f(x_1 , x_2) = 6 x_1 x_2^2 \quad 0 \le x_1 \le l, 0 \le x_2 \le l$

0 elsewhere

Then the marginal probability density $f_1 (x_1)$ is

a. $2x_1$ b. $1 + 3x_1^2$ c. $3x_1^2$ d. $1 - 3x_1^2$ e. $1 - 2x_1$

16. Suppose that X_1 and X_2 are continuous with joint probability density

$$f(x_1, x_2) = \frac{1}{4}(x_1, + x_2) \quad \text{for } 0 \le x_2 \le x_1 \le 2$$

It is easily checked that the marginal probability densities are

$$f_1 (x_1) = \frac{3}{8} x_1^2 \quad \text{and} \quad f_2 (x_2) = \frac{1}{2} + \frac{1}{2} x_2 - \frac{3}{8} x_2^2$$

Using the information

a. Sketch the region on which $f(x_1 x_2) \ge 0$

b. Find $(X_2 \leq \frac{1}{2} \mid X_1 = 1)$

c. Give the integrated integral whose value is $P\left(X_2 > 1, X_1 > \frac{1}{2}\right)$