Exam. III, Math. 323, Fall, 1997 Your name:

This exam. consists of 10 questions. Be sure to show your work. Partial credit may be given if the answer is not correct, and full credit may not be given for a correct answer which is not supported by correct work. Work in the space beside the questions, and mark your answers there. The numbered spaces below are for scoring, not for answers.

Do not use your calculator.

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

1. Suppose X and Y are discrete, with joint p.d.f. $f_{XY}(0,0) = .3$, $f_{XY}(0,1) = .1$, $f_{XY}(1,0) = .1$, $f_{XY}(1,1) = .3$, and $f_{XY}(2,2) = .2$. What is $P(X = Y)$?

(a) .6 (b) .7 (c) .8 (d) .9 (e) cannot be determined

2. Suppose X and Y are jointly distributed with $f_{X,Y}(x,y) = 3xy(x+y)$, for $0 < x < 1$ and $0 < y < 1$. What is the value of the marginal p.d.f. $f_X(x)$, for $0 < x < 1$?

(a) $\frac{3}{2} x^2 + x$ (b) $3x^2$ (c) $x^2 + \frac{4}{3} x$ (d) $2x$ (e) $\frac{4}{3}(x^3 + x)$

3. Suppose X and Y have uniform p.d.f. on the region where $0 < x < 1$ and $0 < y < 1$. What is $P(\frac{X}{Y} \leq 2)$?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) $\frac{4}{5}$

4. Suppose X and Y have joint density which is constant on the region bounded by the lines $x = 0$, $y = 0$, and $x+y = 4$. Are X and Y independent ? Justify your answer.

[Hint: Is $P(X > 3 \ \& \ Y > 3) = P(X > 3) \cdot P(Y > 3)$?]

5. Suppose X and Y have joint p.d.f. $f_{XY}(x,y) = (.5)xe^{-xy}$, for $0 < x < 2$, $y > 0$. What is the value of $f_{Y|x}(y)$ (for $0 < x < 2$ and $y > 0$) ?

(a) $(.5)xe^{-xy}$ (b) xe^{-xy} (c) $(1.5)xe^{-xy}$ (d) $2xe^{-xy}$ (e) $(2.5)xe^{-xy}$

6. If X has p.d.f. $f_X(x) = e^{-x}$ for $x > 0$, and $Y = 2X$, what is the value of the c.d.f. $F_Y(y)$, for $y > 0$?

(a) $2e^{-2y}$ (b) $1 - e^{-2y}$ (c) $2e^{-.5y}$ (d) $1 - e^{-.5y}$ (e) $.5\, e^{-.5y}$

7. Suppose X_1, X_2, X_3, X_4 are independent, all uniform on (0,1). Recall the order statistics X_1', X_2', X_3', X_4'. What is $P(X_1' \leq .3)$?

(a) $(.3)^4$ (b) $1 - (.3)^4$ (c) $1 - (.7)^4$ (d) $(.7)^4$ (e) $4(.3)^7$

8. In the experiment of rolling a fair die 2 times, let X = number of 1's, Y = number of 2's. What is $f_{Y|0}(2)$?

(a) .04 (b) .05 (c) .06 (d) .07 (e) .08

9. If X and Y each have binomial distribution with parameters n = 10 and p = .2, what is E(X + Y) ?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

10. If X is uniform on (0,2), what is $E(X^2)$?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) 1 (e) $\frac{4}{3}$