Final, Math. 323, Fall, 1997 Your name:

This exam. consists of 20 questions. Be sure to show your work. Partial credit may be given if the answer is not correct, and full credit may not be given for a correct answer which is not supported by correct work.

Work in the space beside the questions, and mark your answers there. The numbered spaces below are for scoring, not for answers.

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1. Suppose 10 Mathematics majors, 10 Chemistry majors, and 20 Finance majors sign up for interviews with Miles Laboratories. If the interviewer chooses three at random, what is the probability that all will be Mathematics majors ?

(a) $\frac{10 \cdot 9 \cdot 8}{40 \cdot 39 \cdot 38}$ (b) $\frac{10 \cdot 10 \cdot 20}{40 \cdot 39 \cdot 38}$ (c) $\frac{3}{10}$ (d) $\frac{1}{4}$ (e) $\frac{1}{10 \cdot 10 \cdot 20}$

2. Suppose you toss a fair coin twice. What is the probability that you will get two heads, given that you get heads at least once ?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) $\frac{3}{4}$

3. A company has two plants producing computer monitors. Plant I produces 40 % of the monitors, and among these, 5 % are defective. Plant II produces 60 % of the monitors, and among these, 10 % are defective. If a monitor from the company is found to be defective, what is the probability that it was produced at Plant II ?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) $\frac{5}{6}$

4. If A and B are independent events such that P(A) = .6 and P(B) = .4, what is P(A∪B) ?

(a) .75 (b) .76 (c) .77 (d) .78 (e) .79

5. If you roll a fair die until it you get a 1, what is the probability that this will take at most three rolls ?

(a) $\frac{87}{216}$ (b) $\frac{11}{27}$ (c) $\frac{89}{216}$ (d) $\frac{5}{12}$ (e) $\frac{91}{216}$

6. If you roll a fair die 4 times, what is the probability that you will get two 1's and two 3's ?

(a) $\frac{1}{212}$ (b) $\frac{1}{213}$ (c) $\frac{1}{214}$ (d) $\frac{1}{215}$ (e) $\frac{1}{216}$

7. Suppose X has exponential distribution with $f_X(x) = e^{-x}$ for $x > 0$. If $Y = X^2$, find the value of the c.d.f. $F_Y(y)$, for $y > 0$.

(a) $\sqrt{y}$ (b) $e^{-\sqrt{y}}$ (c) $1 - e^{-\sqrt{y}}$ (d) $1 - \frac{1}{y}$ (e) $1 - \frac{1}{|y|}$

8. Suppose $f_{X,Y}(x,y) = 2$ for (x,y) in the region bounded by lines $x = 0$, $y = 0$, and $x+y = 1$. What is the value of the marginal density for X, $f_X(x)$ for $0 < x < 1$?

(a) $2 - 2x$ (b) 1 (c) $2x$ (d) $\frac{1}{3}\ x^2$ (e) $4 - 3x^2$

9. Let $f_{X,Y}(x,y)$ be as in Problem 8. What is the conditional density $f_{Y|.3}(y)$?

(a) .3, for $0 < y < .3$ (b) 1, for $0 < y < 1$ (c) $\frac{10}{7}$, for $0 < y < .7$

(d) $2y$, for $0 < y < 1$ (e) $2 - 2y$, for $0 < y < .3$

10. Suppose X and Y are discrete, with joint p.d.f.
$f_{X,Y}(0,0) = f_{X,Y}(1,1) = f_{X,Y}(2,2) = .3$, $f_{X,Y}(0,1) = .1$. Find E(X).

(a) .5 (b) .6 (c) .7 (d) .8 (e) .9

11. Let $X_1,...,X_8$ all have uniform distribution on [0,1], and let $Y = X_1+...+X_8$.
What is E(Y) ?

(a) .5 (b) 1 (c) 2 (d) 3 (e) 4

12. Suppose X is continuous, with p.d.f. $f_X(x) = 2x$, for $0 < x < 1$. What is Var(X) ?

(a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{1}{6}$ (d) $\frac{2}{9}$ (e) $\frac{5}{18}$

13. Suppose X has values 0 and 1 with $f_X(0) = .4$ and $f_X(1) = .6$. What is the moment generating function $M_X(t)$?

(a) $.4t + .6$ (b) $.4e^{.6t}$ (c) $e^{-.4t} + e^{-.6t}$ (d) $.4 + .6e^{t}$ (e) $.6e^{-.4t}$

14. Suppose X is a random variable with moment generating function $M_X(t) = \frac{1}{(1-t)^2}$, for $t < 1$. Find the mean and variance of X.

(a) $\mu = 1, \sigma^2 = 2$ (b) $\mu = \sigma^2 = 2$ (c) $\mu = 2, \sigma^2 = 4$ (d) $\mu = 4, \sigma^2 = 6$ (e) $\mu = \sigma^2 = 6$.

15. Suppose X is a random variable with variance 4. What is the best that you can say about the probability p that X differs from its mean by less than 3, using Chebyshev's inequality ?

(a) $p \geq \frac{5}{9}$ (b) $p \leq \frac{5}{9}$ (c) $p \geq \frac{2}{3}$ (d) $p \leq \frac{2}{3}$ (e) $p \leq 1$

16. Let X be the number of defective flashbulbs in a sample of 10 from chosen at random from a production line. Which kind of density function would best model X ?

(a) binomial (b) hypergeometric (c) Poisson (d) exponential (e) normal

17. Let X be the number of calls to a towing service in a given week. Which kind of density function would best model X ?

(a) binomial (b) hypergeometric (c) Poisson (d) exponential (e) normal

18. Let X be the height of a randomly chosen student. Which kind of density function would best model X ?

(a) binomial (b) hypergeometric (c) Poisson (d) exponential (e) normal

19. Let X be the number of 6's in 180 rolls of a fair die. Let F_Z be the c.d.f. for the standard normal. Which of the following is the best approximation of $P(25 \le X \le 35)$?

(a) $P(-.5 \le Z \le .5)$ (b) $P(-.7 \le Z \le .7)$ (c) $P(-.9 \le Z \le .9)$
(d) $P(-1.1 \le Z \le 1.1)$ (e) $P(-1.3 \le Z \le 1.3)$

20. If a certain kind of car needs repair once a year, on the average, what is the probability that a car of this kind will need repair more than 2 times in a given year ?

(a) $5e^{-2}$ (b) $1 - 2.5e^{-1}$ (c) $1 - 2e^{-5}$ (d) $2.5e^{-5}$ (e) $3e^{-2}$