A random variable $X$ is uniformely distributed over the interval $[-1,1]$

1. Which of the following is a graph of the density function of the random variable?

## Answers:


2. What is the probability $P\left(X \leq \frac{1}{2}\right)$ ?
3. What is the probability $P(X>0)$ ?
4. What is the expectation $E(X)$ ?
5. What is the variance $V(X)$ ?

Answers to questions 3-5:
(a) $-\frac{1}{2}$,
(b) 0 ,
(c) $\frac{1}{3}$,
(d) $\frac{1}{2}$,
(e) $\frac{3}{4}$

## Student's name

The following is a $\mathrm{Q}-\mathrm{Q}$ plot for a certain random variable

6. Is the normal distribution a good model?
Answers:
(a) Yes,
(b) No
7. What is the expectation of $X$ ?
8. What is the standard deviation of $X$ ?

Answers to questions 7 and 8:
(a) -10 ,
(b) 0 ,
(c) 10 ,
(d) 20
(e) 35
$\qquad$

A random variable $X$ has a normal distribution with mean 1 and standard deviation 2
9. Find a value $x$, such that $P(X \leq x)=0.6$.

Answers (Values rounded to one decimal place):
(a) 0.3
(b) 0.6
(c) 1.5
(d) 2.0
(e) 2.5
10. Find the probability $P(0.5 \leq X \leq 1.5)$.

Answers (Values rounded to three decimal places):
(a) 0.013,
(b) 0.197
(c) 0.705
(d) 0.922
(e) 1.016
11. The life length of a gadget follows an exponential distribution with mean 10 years. Find the probality that the gadget lasted less then 7 years.

Answers (Values rounded to three decimal places):
(a) 0.012
(b) 0.132
(c) 0.503
(d) 0.667
(e) 0.951

The moment generating function a random variable $X$ is $M(t)=e^{t+t^{2}}$.
12. Find the expected value $E(X)$.
13. Find the variance $V(X)$.

Answers to questions 12-13:
(a) -2
(b) 0
(c) 1
(d) 2
(e) 3
14. What kind of distribution does $X$ have?
(a) uniform, (b) exponential, (c) normal, (d) other, (e) it is not a moment generating function

The joint density function of two random variables $X_{1}$ and $X_{2}$ is given by the formula

$$
f\left(x_{1}, x_{2}\right)=\left\{\begin{array}{cc}
2\left(1-x_{2}\right) & \text { for } 0 \leq x_{1} \leq 1 \text { and } 0 \leq x_{2} \leq 1 \\
0 & \text { elsewhere } .
\end{array}\right.
$$

15. Find the probability $P\left(X_{1}>X_{2}\right)$.
16. Find the conditional probability $P\left(X_{1}>X_{2} \left\lvert\, X_{2}=\frac{1}{2}\right.\right)$.
17. Find the probability $P\left(X_{1}>\frac{1}{2}\right)$.
18. Find the mean $E\left(X_{2}\right)$.

Answers to questions 15-18:
(a) (b)
$\frac{1}{3}$
(c) $\frac{1}{2}$
(d) $\frac{2}{3}$
(e) $\frac{3}{4}$

The random variables $X_{1}$ and $X_{2}$ have means 2 and 1 and variances 4 and 9 respectively. Their coefficient of correlation is 0.5
19. Find the covariance $\operatorname{cov}\left(X_{1}, X_{2}\right)$.
20. Find the expected value of the product $E\left(X_{1} X_{2}\right)$.
21. Find the variance of the difference $V\left(X_{1}-X_{2}\right)$.

Answers to question 17-19:
(a) -2
(b) 1
(c) 3
(d) 5
(e) 7

