

Student's name:.....

1. The process of simultaneously throwing 2 perfect coins is modeled on a sample space $\{s_1, s_2, s_3\}$, Where s_1 represents 2 heads, s_2 represents a head and a tail, and s_3 represents 2 tails. What is the proper assignment of probabilities (p_1, p_2, p_3) to the events s_1, s_2, s_3 ?

Answers:

- (a) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ (b) $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ (c) $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$ (d) $(\frac{1}{3}, \frac{1}{2}, \frac{1}{3})$ (e)
 $(\frac{1}{4}, \frac{1}{3}, \frac{1}{4})$

Among blood donors at a blood bank 35% have O^+ blood, 5% – O^- , 30% – A^+ and 4% – A^- .

2. What is the probability that a randomly chosen donor has type O^- or A^+ ?

3. What is the probability that a randomly chosen donor has none of the types listed above?

Answers to questions 2. and 3:

(a) 0.26

(b) 0.30

(c) 0.35

(d) 0.42

(e) 0.50

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4. Given two events A and B , which of the following is possible?

- I. $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(AB) = \frac{1}{4}$
- II. $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(AB) = \frac{1}{3}$
- III. $P(A) = \frac{3}{4}$, $P(B) = \frac{3}{4}$, $P(AB) = \frac{1}{2}$
- IV. $P(A) = \frac{3}{4}$, $P(B) = \frac{3}{4}$, $P(AB) = 0$.

5. Which of the events in 4. are independent?

Answers to questions 4 and 5:

- (a) I only, (b) I and II, (c) I and III, (d) all of them, (e) none of them

6. Urn A contains two red balls, urn B – one red and one white, urn C – two white ball. A student chooses an urn randomly and is shown that one of the ball in the urn is red. What is the probability that the other ball in the urn is red too.

Answers to question 6.

- (a) $\frac{1}{6}$, (b) $\frac{1}{3}$, (c) $\frac{1}{2}$, (d) $\frac{5}{6}$ (e) $\frac{2}{3}$

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7. 7 integers between 0 and 9 inclusive are chosen randomly to form a 7 digit number. What is the probability that the number is even?

8. What is the probability that an integer formed as in question 7 is divisible by 5?

Answers:

(a) $\frac{1}{9}$, (b) $\frac{1}{5}$, (c) $\frac{1}{4}$, (d) $\frac{1}{3}$, (e) $\frac{1}{2}$

9. In how many ways can three girls and three boys seated on a bench in such a way that no persons of the same sex sit next to each other.

10. Same question for four boys and three girls.

Answers to questions 9 and 10:

(a) $\frac{7!}{4!}$

(b) $4! - 3!$

(c) $3! \cdot 4!$

(d) $2(3!)^2$

(e) $\binom{6}{3}$

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The following table summarizes information about a population of patients suffering from one of three diseases A, B, C.

	Recovered	Dead	All patients
A	70%	30%	40%
B	80%	20%	50%
C	40%	60%	10%

E.g., the table indicates that 80% of patient suffering from B recovered, while 20% died. The total number of patients suffering from B was 50% of all patients.

11. What is the percent of all patients who died?

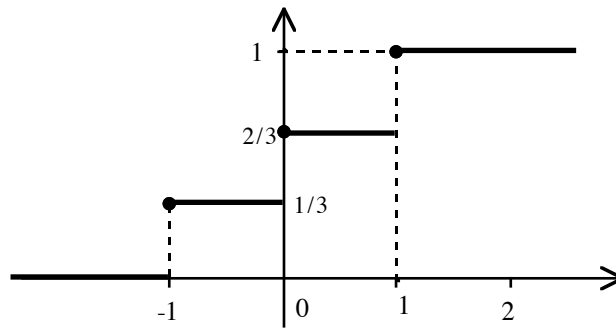
12. What is the probability that a patient who recovered, suffered from disease B?

Answers to questions 11 and 12:

(a) .28 (b) .35 (c) .56 (d) .72 (e) .83

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The figure below represents the graph of the distribution function of a discrete random variable X



13. What values does the random variables take on?

Answers:

(a) all integers, (b) positive integers, (c) $\{-1, 1, 2\}$, (d) $\{-1, 0, 1\}$, (e) $\{-1, 0, 1, 2\}$

14. What is the probability $P(X \leq 2)$?

15. What is the probability $P(X = 2)$?

16. What is the probability $P(X > 0)$?

Answers to questions 14 - 16:

(a) 0

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$,

(d) $\frac{2}{3}$

(e) 1

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A random variable, whose values are positive integers, has a distribution $P(Y = y) = \left(\frac{1}{2}\right)^y$.

17. What is the expected value of Y ?

18. What is the variance of Y ?

Answers to questions 17 and 18:

(a) 0 (b) 1 (c) 1.5, (d) 2 (e) 2.5

A person draws a card from a deck of 52 cards with 4 aces among them. If the card is not an ace, he returns it to the deck, shuffles and draws again until he gets an ace.

19. What is the probability that he will get an ace in no more than two drawing?

20. What is the expected value of the number of drawings.

21. What is the variance of the number of drawings?

Answers to questions 19 - 21:

(a) $\frac{2}{13^2}$

(b) $\frac{25}{13^2}$

(c) $\frac{2}{13}$,

(d) 13

(e) 156

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A random variable X has the moment generating function $M(t) = \frac{1}{1-t}$.

22. Find the expected value of X .

23. Find the variance of X .

24. Find the expected value of the random variable $X - X^3$.

Answers to questions 22 - 24:

(a) -5

(b) 0

(c) 1,

(d) 4

(e) 7

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A balanced coin is thrown 10,000 times.

25. What is the expected value of the number of heads

26. What is the standard deviation?

Answers to questions 25 and 26:

- (a) 50 (b) 1000 (c) 2500 (d) 5000 (e) 7500

27. What is the probability that the number of heads exceeds 600?

Answers (rounded to two decimal places):

(a) 0.01

(b) 0.02

(c) 0.05

(d) 0.1

(e) 0.3

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X_1 and X_2 are two normally distributed random variables, $E(X_1) = 2$, $V(X_1) = 1$, $E(X_2) = 3$, $V(X_2) = 4$.

28. Assuming that the random variables are independent, find $P(X_1 \leq 3, X_2 \leq 5)$.

Answers (Rounded to two decimal places):

(a) 0.17 (b) 0.37 (c) 0.55 (d) 0.71 (e) 0.87

29. Assuming that the random variables are independent, find $E(X_1 X_2)$.

30. Assuming that the coefficient of correlation is 0.5, find $E(X_1 X_2)$.

31. Assuming that the coefficient of correlation is 0.5, find $V(X_1 + X_2)$.

Answers to questions 29-31:

(a) -1 (b) 5 (c) 6 (d) 7 (e) 9