MATH 323. TEST I

NAME:

Directions: You may use your own calculator and your own textbook. You may also use a summary (one side of an 8.5"x11" sheet of paper with notes in your writing). You may use nothing else. You may not pass a calculator, textbook or summary to another person. To receive full credit you must show all your work. Erase or cross out any work you do not want graded.

1.(10 points) In a computer lab 2 of 10 disk drives are defective. If 3 of the drives are chosen at random and tested find the probability that at least one of them is defective.

2.(15 points) The population of a certain city is 40% male and 60% female. Suppose that 50% of the males smoke and 30% of the females smoke. Find the probability that a smoker is male.

3.(10 points) A fair coin is tossed 5 times. Find the probability that there are 2 heads.

4.(10 points) Memory chips are removed from obsolete computers and put into a large bin. Forty percent do not work in the new computers. Find the probability that you must try 3 chips to get 2 that work in a new model.

5.(10 points) A shooter makes 70% of free throws. On the second throw of two, he makes 80% if the first one was made, and 60% if the first one was missed. What is the probability that he makes the second throw?

6.(15 points) For a random variable X, the probability function is given by $p(x) = 0.25,\ 0.5,\ 0.25,$ for $x = -1,\ 0,\ 1$ respectively. Find the standard deviation of X.

7.(10 points) Suppose A and B are independent events such that $P(A \cup B) = 0.4$ and $P(A) = 0.2$. Find $P(B)$.

8.(20 points) Three players a, b, c take turns at a game according to the following rules. At the start a and b play while c is out. The loser is replaced by c and the winner plays against c while the loser is out. The match continues in this way until a player wins twice in succession, thus becoming the winner of the match. (Assume that ties are impossible in the individual games.) The possible outcomes of the match are indicated by the following scheme:

$$aa,\ acc,\ acbb,\ acbaa,\ acbacc,\ acbacbb,\ acbacbaa,\ \ldots$$
$$bb,\ bcc,\ bcaa,\ bcabb,\ bcabcc,\ bcabcaa,\ bcabcabb,\ \ldots$$

In addition, there is the possibility that no player ever wins twice in succession, so the play continues indefinetely according to one of the following patterns

$$acbacbacb\ldots,\ bcabcabca\ldots$$

Assume that at each trial each of the two players has probability 0.5 of wining the game.

a) Show that a wins the match with probability $5/14$. (**Hint:** Assign first probabilities to each outcome of the sample space described above.)

b) What is the probability that c wins the match?

c) Let X be the number of the game at which the match ends. Find $P(X = k)$ for $k \geq 2$, and then find the expected value of X.