MATH 323. TEST II

NAME:

Directions: You may use your own calculator and your own textbook. You may also use a summary (one side of an 8.5"x11" sheet of paper with notes in your writing). You may use nothing else. You may not pass a calculator, textbook or summary to another person. To receive full credit you must show all your work. Erase or cross out any work you do not want graded.

1.(15 points) Let X be a random variable uniformly distributed over the interval $[0, 5]$. Find the number x such that $P(X > x|X > 2) = 1/3$.

2.(10 points) Assume a random variable X has the moment generating function $M(t) = \left(\frac{1}{3}\, e^t + \frac{2}{3}\right)^4$. Find the expected value of X.

3.(15 points) Fix $\theta > 0$ and let $f(x) = \dfrac{1}{2\theta}\, e^{-|x|/\theta}$, $-\infty < x < \infty$.

a) Show that f is a probability density function. This is called the *bilateral exponential density* with parameter θ .

b) If X is a continuous random variable with density function f given above, find the mean and the variance of X.

4.(15 points) If X is a normal random variable with mean 400 and standard deviation 60 find the following (by using tables):
a) $P(X > 490)$;
b) $P(X < 520|X > 490)$.

5.(15 points) The life of a certain kind of light bulb is distributed exponentially, with an average life of 1000 hours.
a) What is the probability that one of these light bulbs, chosen at random, will last at least 500 hours?
b) If you buy 10 of these light bulbs, how many can you expect to last at least 500 hours?

6.(15 points) Suppose the number of cars which stop at a gas station (which is open 24 hours a day) is modeled by a Poisson random variable, with an average of 10 cars per hour.
a) What is the probability that at most two cars will stop at the gas station during the next hour?
b) Let X be the time at which the third car stops at the gas station, measured in hours with respect to a reference time $t = 0$. Find $E(X)$ and $V(X)$.

7.(15 points) Assume that the random variable X is uniformly distributed over the interval $[1, 2]$. If $Y = X^3$, find the density function g of Y.