

MATH 323. TEST II (Make-up)

**NAME:**

Directions: You may use your own calculator and your own textbook. You may also use a summary (one side of an 8.5"x11" sheet of paper with notes in your writing). You may use nothing else. You may not pass a calculator, textbook or summary to another person. To receive full credit you must show all your work. Erase or cross out any work you do not want graded.

1.(10 points) Assume a random variable  $X$  has the probability generating function  $P(t) = 2t^4 - 3t^2 + t + 1$ . Find the expected value of  $X$ .

2.(15 points) Let  $X$  be an exponential random variable with parameter  $\theta = 1$ . Find the number  $x$  such that  $P(X > x | X > 1) = e^{-2}$ .

3.(15 points) If  $X$  is a normal random variable with mean 100 and standard deviation 20 find the following (by using tables):

a)  $P(X < 140)$  ;

b) Find the number  $x$  such that  $P(X > x) = 0.67$  .

4.(15 points) The life of a certain kind of electronic component is distributed exponentially, with an average life of 2000 hours.

a) What is the probability that one of these components, chosen at random, will last at least 1500 hours?

b) A system made of 5 such components fails to function if at least one component is not working. What is the probability that the system will function less than 1500 hours ?

5.(15 points) Suppose the number of customers which arrive at a counter is modeled by a Poisson random variable, with an average of 3 customers per minute.

a) What is the probability that at least one customer will stop at the counter during the next minute?

b) Let  $X$  be the time at which the sixth customer stops at the counter, measured in minutes since the counter opens. Find  $E(X)$  and  $V(X)$ .

6.(15 points) Assume that the random variable  $X$  is uniformly distributed over the interval  $[0, 25]$ . If  $Y = \sqrt{X}$ , find the density function  $g$  of  $Y$ .

7.(15 points) Let  $X$  be a continuous random variable with density  $f(x) = kx(1 - x)$  for  $0 \leq x \leq 1$ , and  $f(x) = 0$  elsewhere.

a) Find the value of  $k$  which makes  $f$  a probability density function.

b) Find the mean and the variance of  $X$ .