NAME:

Directions: You may use your own calculator and your own textbook. You may also use a summary (one side of an 8.5"x11" sheet of paper with notes in your writing). You may use nothing else. You may not pass a calculator, textbook or summary to another person. To receive full credit you must show all your work. Erase or cross out any work you do not want graded.

1.(20 points) Let X and Y be random variables with joint density function

$$f(x, y) = \begin{cases} \frac{3}{4} y & \text{, if } x \geq 0 \,,\, y \geq 0 \,,\, 0 \leq x + y \leq 2 \,, \\ 0 & \text{, elsewhere.} \end{cases}$$

Are X and Y independent? Prove your assertion.

2.(10 points) Let X and Y be independent random variables with moment generating functions $M_X(t) = 1 + t$ and $M_Y(t) = e^t$. Find the moment generating function for $U = 2X + Y$, and use it to compute $E(U)$.

3.(20 points) Let X and Y be random variables with joint density function

$$f(x,y) = \begin{cases} 1 & , \text{ if } 0 \le x \le 2 \,,\, 0 \le y \le 1 \,,\, 2y \le x \\ 0 & , \text{ elsewhere} \end{cases}$$

a) Find the marginal density $f_X(x)$ of X.

b) Find $P(-1 \le Y \le 1/4 \,|\, X = 1)$.

c) Compute $E(Y \,|\, X = x)$ as a function of x .

2

4.(20 points) Assume X and Y are jointly uniformly distributed over the unit square $[0,1] \times [0,1]$ and let $U = Y/X$. Find the density function $f_U(u)$ of U.

5.(15 points) Let X_1 and X_2 be random variables such that $E(X_1) = E(X_2) = 2$, $E(X_1^2) = E(X_2^2) = 7$, and $E(X_1 X_2) = 3$. Find $V(X_1 - 2X_2)$.

6.(15 points) Suppose ten people each toss two coins. Let Y_1 denote the number of people who got two tails, Y_2 the number of people who got two heads, and Y_3 the number of people who got a head and a tail.

a) Find $P(Y_1 = 2, Y_2 = 3, Y_3 = 5)$.

b) Find $V(2Y_1 + 5Y_3)$.