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Mathematics 323: Introduction to Probabilities
Spring Semester 1998
Final Exam
May 4, 1998

This Examination contains 25 questions, worth 6 points each, for a maximum score of 150. Fill in your answers on this cover sheet by placing an X through one letter for each problem. You may use your own calculator and your own textbook. You may also use a summary (an 8.5”x11” sheet of paper with notes in your writing). You may use nothing else. You may not pass a calculator, textbook or summary to another person.

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
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21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e

Sign the pledge:

“On my honor, I have neither given nor received unauthorized aid on this Exam.”

Signature: ______________________________

GOOD LUCK

1. Given the moment generating function $M_X(t) = (1-2t)^{-1/2}$, find $E(X^2)$.

(a) 3 (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) $\frac{5}{2}$

2. A poorly done manuscript 100 pages long is found to have 300 misprints. If a page is selected at random, what is the probability of finding fewer than the expected number of misprints?

(a) $\frac{1}{300}$ (b) $e^{1/3}$ (c) $8.5e^{-3}$ (d) $100\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{98}$ (e) 0

3. A grocer buys facial tissue once a month. The number of packages that he sells in any month is normal with mean 1600 and standard deviation 40. Find the smallest number of packages he can buy at the beginning of the month to be 95% sure he does not run out.

(a) 1700 (b) 1666 (c) 1633 (d) 1680 (e) 1640

4. If X is normal with parameters $\mu = 1$ and $\sigma = 3$, what is $E(X^2)$?

(a) 4 (b) 8 (c) 6 (d) 10 (e) 2

5. Use the normal distribution to approximate the probability of getting at most 25 "fives" in 180 tosses of a fair die.

(a) 0.8413 (b) 0.2119 (c) 0.1024 (d) 0.5793 (e) 0.1841

6. An archer hits the bulls-eye in a target 30% of the time. The probability that it will take 13 shots to make 9 bulls-eyes is

(a) $(0.3)^9(0.7)^4$ (b) $495(0.3)^9(0.7)^4$ (c) $715(0.3)^9(0.7)^4$ (d) $715(0.3)^8(0.7)^4$ (e) $495(0.3)^8(0.7)^4$

7. Events A and B have probabilities 0.5 and 0.4 respectively, and $P(AB) = 0.2$. What is $P(A \cup B)$?

(a) 0.6 (b) 1 (c) 0.7 (d) 0.8 (e) 0.9

8. If $f(x) = cx$ for $0 < x < 1$ and $f(x) = 0$ elsewhere, what must c be in order for $f(x)$ to be a probability density function?

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) 3 (e) $\frac{1}{3}$

9. If $P(A) = 0.46$, $P(A \mid B) = 0.4$, $P(AB) = 0.36$, then $P(A \cup B)$ is

(a) 1 (b) 0.72 (c) 0.96 (d) 0.64 (e) 0.84

10. A bag contains 3 coins: 2 ordinary nickels, and one 2-headed nickel (heads on both sides). One of the 3 coins is selected at random and it is flipped. The flip shows a head. What is the probability that the 2-headed coin was selected?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{5}{6}$ (e) $\frac{7}{12}$

11. Say A, B, C are events for which $P(A) = 0.1$, $P(B) = 0.2$, $P(C) = 0.3$, $P(AB) = 0.03$, $P(AC) = 0.03$, $P(BC) = 0.05$. Which of the following is true?

(a) A and B are independent (b) B and C are independent (c) No two of A, B, C are independent
(d) A and C are independent (e) You really shouldn't choose this one

12. Smith and Jones will take turns flipping a fair coin until a head appears. Smith has the first toss. What is the probability that Smith tosses the first head?

(a) $\frac{7}{12}$ (b) $\frac{5}{6}$ (c) $\frac{11}{12}$ (d) $\frac{3}{4}$ (e) $\frac{2}{3}$

13. The number of 5-card poker hands which are four-of-a-kind (out of a deck of 52 cards) is

(a) 13 (b) 13×52 (c) $13^4 \times 48$ (d) 48 (e) 13×48

14. A wallet contains 5 pennies and 5 nickels. Three coins are taken at random. Find the probability that exactly 2 nickels were taken.

(a) $\frac{1}{2}$ (b) $\frac{7}{12}$ (c) $\frac{2}{3}$ (d) $\frac{5}{12}$ (e) $\frac{1}{3}$

15. Let $f_{X,Y}(x, y) = 4xy$ for $0 \leq x \leq 1$, $0 \leq y \leq 1$, and 0 elsewhere. Find $P(X < 12, Y < \frac{1}{2})$.

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$ (e) 0

16. Let X, Y be jointly uniform for $0 \leq x \leq 2$, $0 \leq y \leq 3$. Find $P(Y \geq X)$.

(a) $\frac{2}{3}$ (b) $\frac{5}{6}$ (c) $\frac{5}{12}$ (d) $\frac{8}{9}$ (e) $\frac{1}{2}$

17. Let $f_X(x) = \frac{1}{2}x$ for $0 \leq x \leq 2$, and 0 elsewhere. If $U = X^3$ then, for $0 \leq u \leq 8$, $f_U(u)$ is

(a) $\frac{1}{2}u^{1/3}$ (b) $\frac{1}{3}u^{-2/3}$ (c) $\frac{1}{6}u^{-1/3}$ (d) $\frac{1}{8}u^3$ (e) 0

18. A die is rolled until the first "six" appears. If X denotes the number of the roll on which the first "six" appears, then $P(X = 3)$ is

(a) $\frac{5}{36}$ (b) $\frac{25}{108}$ (c) $\frac{25}{216}$ (d) $\frac{125}{216}$ (e) $\frac{1}{216}$

19. If X is exponential with mean $\frac{1}{2}$, then $P(X > 7 \,|\, X > 3)$ is

(a) e^{-4} (b) e^{-7} (c) e^{-2} (d) e^{-8} (e) $e^{-3} - e^{-7}$

20. Suppose $V(X_1 - 2X_2) = 9$, $V(X_1) = 3$, $V(X_2) = 1$. Find $cov(X_1, X_2)$.

(a) $\frac{1}{2}$ (b) -1 (c) $-\frac{5}{2}$ (d) 2 (e) $-\frac{1}{2}$

21. Let X_1, X_2 be independent, each uniformly distributed on the interval $(-1, 1)$. If $U = \max\{X_1, X_2\}$ find the density $f_U(u)$ of U for $-1 < u < 1$.

(a) $\frac{1}{2}$ (b) $\frac{3}{8}(u+1)^2$ (c) $\frac{1}{2}(u+1)$ (d) $u + \frac{1}{2}$ (e) $2u + 1$

22. Let $f(x_1, x_2) = 15x_1x_2^2$, for $0 \leq x_2 \leq x_1 \leq 1$, and 0 elsewhere, be the joint density of X_1, X_2. Find $P(X_2 \geq \frac{1}{4} \,|\, X_1 = \frac{1}{2})$.

(a) 0.9 (b) 0.8 (c) 0.75 (d) 0.85 (e) 0.875

23. Given the joint density for X_1, X_2, $f(x_1, x_2) = 1$ for $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 2$, $0 \leq 2x_1 + x_2 \leq 2$, and 0 elsewhere, find $E(X_1 \,|\, X_2 = 1)$.

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) 3 (d) 1 (e) $\frac{1}{2}$

24. Let X_1, X_2 be jointly uniform on $(0,1)\times(0,1)$. If $U=\sqrt{X_1X_2}$ find the density $f_U(u)$ of U for $0<u<1$.

(a) $1-u^2$ (b) $2-2u$ (c) $-2\ln(1-u)$ (d) $-4u\ln u$ (e) $3u^2$

25. Let $X_1,\ X_2,\ \ldots,\ X_{75}$ be independent random variables, each of them uniform on $(-1,1)$. Using the Central Limit Theorem, approximate $P(X_1+X_2+\ldots+X_{75}>0.3)$.

(a) 0.5479 (b) 0.4201 (c) 0.4582 (d) 0.4761 (e) 0.5152