MATH 323. TEST III

NAME:

Directions: You may use your own calculator and your own textbook. You may also use a summary (one side of an 8.5"x11" sheet of paper with notes in your writing). You may use nothing else. You may not pass a calculator, textbook or summary to another person. To receive full credit you must show all your work. Erase or cross out any work you do not want graded. You are taking this exam under the honor code.

1.(20 points) Let X and Y be random variables with moment generating functions

$$M_X(t) = \frac{1}{1-2t} \quad , \quad M_Y(t) = e^{t+t^2} \ .$$

Find the expected value of $U = 3X + 2Y$.

2.(20 points) The random variable X has a beta distribution with parameters $\alpha = \beta = 1/2$. Find the variance of $U = X^2$.

3.(20 points) A certain experiment has 3 possible outcomes, A, B and C, which occur with probabilities $P(A) = 0.1$, $P(B) = 0.2$, $P(C) = 0.7$. The experiment is repeated 10 times independently. Let Y_1 denote the number of occurences of outcome A and Y_2 denote the number of occurences of outcome B. Find $V(Y_1 + 2Y_2)$.

4.(20 points) Let X_1 and X_2 be continuous random variables with joint density function

$$f(x_1, x_2) = \begin{cases} \frac{3}{4}\, x_1 \,, & \text{if } x_1 \geq 0,\ x_2 \geq 0,\ x_1 + x_2 \leq 2 \,, \\ 0 \,, & \text{elsewhere .} \end{cases}$$

a) Find the density function $f_{X_1}(x_1)$ of X_1.

b) Find $P(1/2 \leq X_2 \leq 2 \mid X_1 = 1)$.

5.(20 points) Assume that X and Y are continuous random variables with joint density function

$$f(x,y) = \begin{cases} \frac{1}{2}\,, & \text{if } 0 < x < 1,\ 0 < y < 2\,, \\ 0\,, & \text{elsewhere}\,. \end{cases}$$

Find the density function of the random variable $U = Y/X$.