Name: ______________________________ **Instructor:** Dan Coman

Math 323 Final Exam, May 10, 2000

You may use your own calculator and your own textbook. You may also use a summary (an 8.5"x11" sheet of paper with notes in your writing). You may use nothing else. You may not pass a calculator, textbook or summary to another person.

Part I consists of 15 multiple choice questions. Record your answers by placing an × through one letter for each problem on this answer sheet. **Part II** consists of 3 partial credit problems. Write your answer and show ALL your work on the page on which the question appears. **Do not remove this answer sheet.** You will return the whole exam.

You are taking this exam under the honor code.

1.	a	b	c	d	e	9.	a	b	c	d	e
2.	a	b	c	d	e	10.	a	b	c	d	e
3.	a	b	c	d	e	11.	a	b	c	d	e
4.	a	b	c	d	e	12.	a	b	c	d	e
5.	a	b	c	d	e	13.	a	b	c	d	e
6.	a	b	c	d	e	14.	a	b	c	d	e
7.	a	b	c	d	e	15.	a	b	c	d	e
8.	a	b	c	d	e						

Multiple choice points: ______________________

Partial credit points: ______________________

Total points: ______________________

Part I: Multiple Choice Questions *(6 points each)*

1. Find $P(A \mid B)$ if $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \cup B) = 0.8$.

(a) $\frac{7}{8}$ (b) $\frac{3}{7}$ (c) $\frac{3}{4}$ (d) $\frac{4}{7}$ (e) $\frac{1}{2}$

2. Assume that the moment generating function of a random variable X is $M(t) = (2e^t - 1)^5$. Find $E(X)$.

(a) 1 (b) 10 (c) 5 (d) 2 (e) -1

3. 10% of the tires in a large lot are defective. 20 tires are seleted at random for a sale. What is the probability that 3 out of these 20 tires are defective?

(a) $1140(0.1)^{17}(0.9)^3$ (b) $(0.1)^3(0.9)^{17}$ (c) $(0.1)^{17}(0.9)^3$

(d) $6840(0.1)^3(0.9)^{17}$ (e) $1140(0.1)^3(0.9)^{17}$

4. For what value of c is the function $f(x) = cx^2$ for $-1 < x < 1$ and $f(x) = 0$ elsewhere a probability density function?

(a) $\frac{2}{3}$ (b) 1 (c) 0 (d) $\frac{3}{2}$ (e) $\frac{1}{2}$

5. If the random variable X is uniformly distributed in $[0, 1]$ find $E(X^2 - X)$.

(a) $\frac{1}{3}$ (b) $-\frac{1}{6}$ (c) $-\frac{1}{2}$ (d) 0 (e) 1

6. If X is a normal random variable with parameters $\mu = 20$ and $\sigma^2 = 25$, find $P(X > 30)$.

(a) 0.0228 (b) 0.5228 (c) 0.2580 (d) 0.4306 (e) 0.1217

7. Suppose Y is a binomial random variable with parameters $n = 300$ and $p = \frac{1}{4}$. Using the normal distribution, approximate the probability $P(68 \leq Y \leq 82)$.

(a) 0.6826 (b) 0.6476 (c) 0.3413 (d) 0.3238 (e) 0.7016

8. If X has an exponential density with mean 100 find $P(X > 100)$.

(a) e^{-100} (b) e^{-10000} (c) e^{-1} (d) 1 (e) $e^{-0.5}$

9. Let X_1, X_2 be independent random variables, both exponential with mean 1. For $u > 0$ the probability density function of $U = \min\{X_1, X_2\}$ is given by:

(a) e^{-u} (b) $2(1 - e^{-u})e^{-u}$ (c) $\frac{1}{2}e^{-u/2}$ (d) $2e^{-2u}$ (e) $3e^{-3u}$

10. The population of a certain city is 40% male and 60% female. Suppose that 50% of the males smoke and 30% of the females smoke. Find the probability that a randomly selected smoker is male.

(a) $\frac{9}{19}$ (b) $\frac{10}{31}$ (c) $\frac{2}{10}$ (d) $\frac{6}{10}$ (e) $\frac{10}{19}$

11. A shooter makes 75% of the free throws. What is the probability that he needs to shoot 5 throws in order to make 3?

(a) $6(0.75)^3(0.25)^2$ (b) $10(0.75)^3(0.25)^2$ (c) $12(0.75)^3(0.25)^2$

(d) $(0.75)^3(0.25)^2$ (e) $15(0.75)^3(0.25)^2$

12. The number of cars which stop at a gas station is modeled by a Poisson random variable with an average of 10 cars per hour. What is the probability that at most one car will stop at this gas station during the next 30 minutes?

(a) $11e^{-10}$ (b) $6e^{-5}$ (c) $5e^{-5}$ (d) $10e^{-10}$ (e) e^{-5}

13. Peter rolls 3 fair dice and adds the numbers on the top face of each die. What is the probability that he will get a sum of 5?

(a) $\frac{1}{216}$ (b) $\frac{1}{72}$ (c) $\frac{1}{36}$ (d) $\frac{1}{108}$ (e) $\frac{5}{216}$

14. Assume that $X_1, X_2, \ldots, X_{100}$ are independent and identically distributed random variables, each with mean $\mu = 0$ and standard deviation $\sigma = 5$. Use the Central Limit Theorem to approximate $P(X_1 + X_2 + \ldots + X_{100} < 25)$.

(a) 0.1915 (b) 1 (c) 0.5987 (d) 0.0987 (e) 0.6915

15. Assume that X_1, X_2 are random variables with

$$E(X_1) = 0,\ V(X_1) = 5,\ E(X_2) = 1,\ V(X_2) = 2,\ E(X_1 X_2) = 3\ .$$

Find $V(X_1 - X_2)$.

(a) 11 (b) 10 (c) 3 (d) 1 (e) 7

Part II: Partial Credit Questions

16.(20 points) Let X_1, X_2 be discrete random variables with joint probability distribution given by:

$$P(X_1 = 0, X_2 = -1) = \frac{4}{12}\ ,\ P(X_1 = 0, X_2 = 0) = \frac{2}{12}\ ,$$

$$P(X_1 = 1, X_2 = -1) = \frac{5}{12}\ ,\ P(X_1 = 1, X_2 = 0) = \frac{1}{12}\ .$$

Find $E(X_1 + X_2)$.

17.(20 points) Assume that X_1, X_2 are continuous random variables with joint density function

$$f(x_1, x_2) = \begin{cases} e^{-x_1}\ , & \text{if } 0 \leq x_2 \leq x_1 < +\infty\ , \\ 0\ , & \text{elsewhere}\ . \end{cases}$$

a) Find the density function $f_{X_1}(x_1)$ of X_1.

b) Compute $E(X_2 \mid X_1 = x_1)$ as a function of x_1.

18.(20 points) Let X, Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} 2x\ , & \text{if } 0 < x < 1,\ 0 < y < 1\ , \\ 0\ , & \text{elsewhere}\ . \end{cases}$$

Find the density function of the random variable $U = e^{XY}$.