Exam. I, Math. 324, Spring, 1998
Your name:

This exam. consists of 11 questions. There is a table of the normal distribution and a sheet of information about basic random variables at the end.

Be sure to show your work. Partial credit may be given if the answer is not correct, and full credit may not be given for a correct answer which is not supported by correct work. Work in the space beside the questions, and mark your answers there. The numbered spaces below are for scoring, not for answers.

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1. Suppose X is the time until the next call to the ND fire dept. What kind of distribution would you assume for X ?

(a) Poisson (b) geometric (c) negative binomial (d) normal (e) exponential

2. A recruiter who conducts job interviews for an investment company has found that 1 in 20 job applicants is fully qualified. Let X be the number of not-fully-qualified applicants interviewed in locating 4 who are fully qualified. What kind of distribution would you assume for X ?

(a) Poisson (b) geometric (c) negative binomial (d) normal (e) Γ

3. Let X be the time (from opening) until the 4th person has come to the computer repair desk. What kind of distribution would you assume for X ?

(a) Poisson (b) geometric (c) negative binomial (d) normal (e) Γ

4. Let X be normal with parameters $\mu = 4$, $\sigma = 2$. Find c such that $P(X > c) \approx .02$.

(a) 8.1 (b) 8.2 (c) 8.3 (d) 8.4 (e) 8.5

5. Let Z be standard normal, and let $Y = Z^2$. Find expressions for $F_Y(y)$ and $f_Y(y)$ for $y > 0$.

(a) $\int_0^y f_Z(z)\,dz$; $f_Z(y)$ (b) $2\int_0^{y^2} f_Z(z)\,dz$; $4yf_Z(y2)$ (c) $\int_0^{y^2} f_Z(z)\,dz$; $f_Z(y^2)$

(d) $2\int_0^{\sqrt{y}} f_Z(z)\,dz$; $(1/\sqrt{y})f_Z(\sqrt{y})$ (e) $2\int_0^{\sqrt{y}} f_Z(z)\,dz$; $2f_Z(\sqrt{y})$

6. If Y has Γ-distribution with density function $f_Y(y) = (1/\sqrt{2\pi y}\,)e^{-(1/2)y}$ for $y > 0$, what are the parameters λ and r ? [Recall: $\sqrt{\pi} = \Gamma(1/2)$]

(a) $\lambda = 1, r = 2$ (b) $\lambda = r = 1/2$ (c) $\lambda = r = 1$ (d) $\lambda = 1/2, r = -1/2$ (e) cannot be determined

7. Suppose W is an estimator for the parameter θ. What does it mean to say that W is <u>unbiased</u> ?

(a) $E(W) = \theta$ (b) $W = \theta$ (c) $E((W - \theta)^2) = 0$

(d) W is independent of θ (e) W has approximately normal distribution

8. Suppose X is geometric with parameter p, and let q = 1-p. Consider W = 1/X as an estimator for p. Which of the following represents E(W) ?

(a) p(1+q+q2+...) (b) $p(1+q/2+q^2/3+...)$ (c) p(1+1/q+1/q2+...)

(d) p(1+2/q+3/q2+...) (e) $p(1-q+q^2-...)$

Say whether W is unbiased.

9. 4. Suppose X is a random variable with standard deviation 4. Let $X_1,...,X_{100}$ be a random sample from X, and let $\bar{X}$ be the sample mean. What is the standard deviation of $\bar{X}$?

(a) .2 (b) .3 (c) .4 (d) .5 (e) cannot be determined

10. Let X be exponential with $f_X(x) = (1/\theta)e^{-(1/\theta)x}$, $x > 0$. Let $X_1,\ldots,X_n$ be a random sample from X. Both $W_1 = \bar{X}$ and $W_2 = X_1$ are unbiased estimators for θ. Which of the estimators is more efficient (for $n > 1$) ?

(a) W_1 (b) W_2 (c) they are equally efficient

11. Let X and Y be independent binomial random variables, where X has parameters m and p, Y has parameters n and p. Both $W_1 = (X+Y)/(m+n)$ and $W_2 = (1/2)(X/m+Y/n)$ are unbiased estimators for p. What is the relative efficiency of W_1 with respect to W_2 ?

(a) $2mn/(m+n)^2$ (b) $(m+n)^2/4mn$ (c) $(m+n)/4mn$ (d) $pq/(m+n)$ (e) $pq(m+n)/4mn$

Special kinds of random variables

Binomial: $f_X(k) = \binom{n}{k} p^k q^{n-k}$ for $k = 0,\dots,n$; $\mu = np$, $\sigma^2 = npq$, $M_X(t) = (pe^t+q)^n$

Poisson: $f_X(k) = e^{-\lambda}\lambda^k/k!$ for $k = 0,1,2,\dots$; $\mu = \sigma^2 = \lambda$, $M_X(t) = e^{\lambda(e^t-1)}$

Geometric: $f_X(k) = pq^{k-1}$ for $k = 1,2,\dots$; $\mu = 1/p$, $\sigma^2 = q/p^2$, $M_X(t) = pe^t/(1-qe^t)$

Negative binomial: $f_X(k) = p^r q^k \binom{r+k-1}{r-1}$ for $k = 0,1,2,\dots$; $\mu = rq/p$, $\sigma^2 = rq/p^2$

Uniform: $f_X(x) = 1/(b-a)$ for $a < x < b$; $\mu = (a+b)/2$, $\sigma^2 = (b - a)^2/12$

Normal: $f_X(x) = (1/\sqrt{2\pi}\,\sigma)e^{-(1/2)((x-\mu)/\sigma)^2}$; $M_X(t) = e^{(\mu t + \sigma^2 t^2/2)}$

Exponential: $f_X(x) = \lambda e^{-\lambda x}$ for $x > 0$; $\mu = 1/\lambda$, $\sigma^2 = 1/\lambda^2$, $M_X(t) = \lambda/(\lambda-t)$

Gamma: $f_X(x) = \lambda^r x^{r-1}e^{-\lambda x}/\Gamma(r)$ for $x > 0$; $\mu = r/\lambda$, $\sigma^2 = r/\lambda^2$, $M_X(t) = (\lambda/(\lambda-t))^r$