Exam. II, Math. 324, Spring, 1998
Your name:

This exam. consists of 10 questions. There is a normal table at the end. Information about density functions, etc., is given as needed.

Be sure to show your work. Partial credit may be given if the answer is not correct, and full credit may not be given for a correct answer which is not supported by correct work. Work in the space beside the questions, and mark your answers there. The numbered spaces below are for scoring, not for answers.

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1. Let Y be Poisson with parameter λ unknown. Let $Y_1,...,Y_n$ be a random sample from Y, and suppose $W = h(Y_1,...,Y_n)$ is an efficient estimator for λ. What is Var(W) ?

<u>Recall</u>: $f_Y(y) = \dfrac{e^{-\lambda}\lambda^y}{y!}$, for y = 0,1,2,..., E(Y) = Var(Y) = λ.

(a) λ (b) λ^2 (c) $\dfrac{\lambda}{n}$ (d) $\dfrac{\lambda^2}{n}$ (e) cannot be determined.

2. Suppose Y has parameter λ, and for each n, W_n is an estimator for λ (based on a random sample of size n) such that $E(W_n) = \lambda$, and $Var(W_n) = \dfrac{\lambda}{n}$. What can you conclude about W_n as an estimator for λ ? Give all correct choices--there may be more than one.

(a) it is unbiased (b) it is consistent (c) it is sufficient (d) it is best
(e) it is the maximum likelihood estimator

3. Let Y be geometric with parameter p. Let $Y_1,...,Y_n$ be a random sample from Y. What is the likelihood function $L(y_1,...,y_n;p)$?

Recall: $f_Y(y) = p(1-p)^{y-1}$, for y = 1,2,3,....

(a) $p^n(1-p)^{(\Sigma y_i) - n}$ (b) $p^{\Sigma y_i}(1-p)^n$ (c) $(p(1-p))^{\Sigma y_i}$ (d) $p^n(1-p)^{-\Sigma y_i}$ (e) $p^{n - \Sigma y_i}$

4. In Problem 3, $L(y_1,...,y_n;p)$ is expressed as g(w), where $w = \Sigma y_i$. What does this say about $W = \Sigma Y_i$ as an estimator for p ?

(a) it is unbiased (b) it is consistent (c) it is sufficient (d) it is best
(e) it is the maximum likelihood estimator

5. Let Y be uniform on $(-\theta,\theta)$. If a sample of size 5 yields values -4,2,3,-2,1, what is the maximum likelihood estimate for θ ?

Recall: For Y uniform on (a,b), $f_Y(y) = \frac{1}{b-a}$ for $a < y < b$, $E(Y) = \frac{a+b}{2}$, $Var(Y) = \frac{(b-a)^2}{12}$

(a) 0 (b) 3 (c) 4 (d) 5 (e) 6

6. For Y as in Problem 5, express θ in terms of $\mu_{(2)}$--$\mu_{(1)} = 0$, and determine a method of moments estimator.

(a) $\frac{\Sigma Y_i}{n}$ (b) $\frac{\Sigma Y_i^2}{n}$ (c) 0 (d) $\sqrt{\frac{3\Sigma X_i^2}{n}}$ (e) $\sqrt{\frac{\Sigma X_i}{12n}}$

7. What is the value of $z_{.025}$?

(a) 1.95 (b) 1.96 (c) 1.97 (d) 1.98 (e) 1.99

8. Suppose Y is normal with $\sigma = 3$, μ unknown. For a random sample $Y_1, \ldots, Y_{100}$, let W be the sample mean. Find ε such that $P(|W - \mu| < \varepsilon) = .95$.

Recall: $f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$, $E(Y) = \mu$, $Var(Y) = \sigma^2$.

(a) $.2z_{.025}$ (b) $.3z_{.025}$ (c) $.4z_{.025}$ (d) $.5z_{.025}$ (e) cannot be determined

9. Among 100 randomly chosen seniors, 90 said that their job prospects were very good. Give a 90% confidence interval estimate of the over-all proportion of seniors who feel this way.

(a) $.9 \pm .03z_{.05}$ (b) $.9 \pm .04z_{.05}$ (c) $.9 \pm .05z_{.1}$ (d) $.9 \pm .06z_{.1}$ (e) $.9 \pm .07z_{.1}$

10. The natural estimator for the parameter p in a Bernoulli experiment, with a sample of size n, is the sample proportion W_n. What sample size should you take to be 90% sure that $|W_n - p| < .01$?

(a) $n \geq 45z_{.05}$ (b) $n \geq (46z_{.1})^2$ (c) $n \geq (48z_{.01})^2$ (d) $n \geq (49z_{.1})^2$ (e) $n \geq (50z_{.05})^2$