Exam. III, Math. 324, Spring, 1998

Your name:

This exam. consists of 10 questions. The test booklet includes normal and χ^2 tables. Information about density functions, etc., is given as needed.

Be sure to show your work. Partial credit may be given if the answer is not correct, and full credit may not be given for a correct answer which is not supported by correct work. Work in the space beside the questions, and mark your answers there. The numbered spaces below are for scoring, not for answers.

1.

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1. The heights of barley plants of a common variety are approximately N(23.0,4.0). A new variety of barley shows promise of being shorter. To test $H_0: \mu = 23.0$ v. $H_1: \mu < 23.0$, a horticulturist plans to examine 100 plants of the new variety and apply a decision rule of the form: Reject H_0 if and only if $\tilde{Y} \leq y^*$. Find the appropriate y^* for a test at the 2% significance level. (Assume that the variance is the same for the two varieties.)

(a) 22.5 (b) 22.6 (c) 22.7 (d) 22.8 (e) 22.9

2. Suppose the horticulturist in problem 1 wishes to determine not whether the new variety of barley is shorter, but whether its height is significantly different from 23.0. Give the appropriate hypotheses H_0 and H_1.

(a) H_0: $\mu = 23.0$ v. H_1: $\mu \neq 23.0$ (b) H_0: $\mu = 23.0$ v. H_1: $\mu < 23.0$
(c) H_0: $\mu = 23.0$ v. H_1: $\mu > 23.0$

3. Suppose we test H_0: $p = 1/2$ v. H_1: $p \neq 1/2$ using a statistic Y which is binomial with $n = 5$, and taking as the decision rule: Reject H_0 if and only if $Y = 5$ or $Y = 0$. What is the probability β of Type II error if $p = 1/3$?

[For binomial Y, $f_Y(y) = \binom{n}{y} p^y q^{n-y}$ for $y = 0,\ldots,n$, $\mu = np$, $\sigma^2 = npq$.]

(a) 68/81 (b) 23/27 (c) 70/81 (d) 71/81 (e) 8/9

4. Let p be the proportion of people from the Southern hemisphere who are born in January-June. To test the hypotheses H_0: $p = 1/2$ v. H_1: $p > 1/2$, researchers question 100 randomly chosen Australians about their birthdays. Let Y be the number having birthdays in January-June. What is the approximate value of $P(Y \geq 53 | H_0)$ (this is the same as $P(Y \geq 52.5 | H_0)$) ?

(a) .23 (b) .25 (c) .27 (d) .29 (e) .31

5. If a sample of size 10 yields $\Sigma X_i = 40$ and $\Sigma X_i^2 = 196$, what is the sample variance s^2 ?

(a) 3 (b) 4 (c) 5 (d) 6 (e) cannot be determined

6. Let $Y_1, Y_2, \ldots$ be a random sample from a random variable Y with mean μ and variance σ^2. Without knowing more about Y, what can you say about the sample mean $\bar{Y}$? Mark all answers which are correct--there may be more than one.

(a) $E(\bar{Y}) = \mu$ (b) $Var(\bar{Y}) = \sigma^2$ (c) for $\varepsilon > 0$, $\lim_{n \to \infty} P(|\bar{Y} - \mu| < \varepsilon) = 1$

(d) $\bar{Y}$ has normal distribution (e) $\lim_{n \to \infty} P\left(\frac{\sqrt{n}(\bar{Y}-\mu)}{\sigma} \in I\right) = P(Z \in I)$

7. Suppose Y has mean 10.0 and variance 25.0. If $Y_1,\ldots,Y_{100}$ is a random sample from Y, what is the approximate value of $P(\tilde{Y} \geq 11.0)$?

(a) .023 (b) .025 (c) .027 (d) .029 (e) .031

8. Let $Y_1,\ldots,Y_8$ be a random sample from $Y \sim N(10,7)$, and let s^2 be the sample variance. Find $Var(s^2)$. Recall that if $U \sim \chi^2_n$, then $Var(U) = 2n$.

(a) 8 (b) 10 (c) 12 (d) 14 (e) 16

9. Let $Z_1,...,Z_{10}$ be a random sample from N(0,1) (standard normal), and let $U = \Sigma Z_i^2$. What is the distribution of U ?

(a) χ_9^2 (b) χ_{10}^2 (c) N(0,10) (d) N(0,1) (e) cannot be determined

10. A random sample of size 10 from a normal population yields sample variance 4.0. Give a 95% confidence interval estimate for the population variance.

(a) (2.70,19.0) (b) $\left(\frac{40}{16.9}, \frac{40}{3.33}\right)$ (c) $\left(\frac{36}{16.9}, \frac{36}{3.33}\right)$

(d) $\left(\frac{36}{19.0}, \frac{36}{2.70}\right)$ (e) $\left(\frac{3.33}{40}, \frac{16.9}{40}\right)$