Final, Math. 324, Spring, 1998

Your name:

This exam. consists of 16 questions. The test booklet includes normal, T, and χ^2 tables. There is also a sheet with information on density functions, means, and variances of familiar kinds of random variables.

Be sure to show your work. Partial credit may be given if the answer is not correct, and full credit may not be given for a correct answer which is not supported by correct work. Work in the space beside the questions, and mark your answers there. The numbered spaces below are for scoring, not for answers.

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1. Suppose X is normal with $\mu = 100$, $\sigma^2 = 16$. Find a such that $P(X > a) \approx .10$.

(a) 104 (b) 105 (c) 106 (d) 107 (e) 108

2. On average, a switchboard receives 49 calls per hour. What is the probability of > 52 calls in an hour ? Use an appropriate normal approximation (with continuity correction).

(a) .27 (b) .29 (c) .31 (d) .33 (e) .35

3. In Problem 2, what kind of distribution would you assume for X = number of calls in a 1-hour period ?

(a) binomial (b) geometric (c) negative binomial (d) Poisson (e) Gamma

4. On average, a switchboard receives 49 calls per hour. What kind of distribution would you assume for Y = time until the 3rd call ?

(a) binomial (b) geometric (c) negative binomial (d) Poisson (e) Gamma

5. On any inspection of a certain restaurant, there is a 30% chance a serious violation of health code will be found. Let X = number of inspections until the 4th serious violation is found. What kind of distribution would you assume for X ?

(a) binomial (b) geometric (c) negative binomial (d) Poisson (e) Gamma

6. Suppose Y is uniform on $[0,\theta]$. Suppose $Y_1,\dots,Y_n$ is a random sample from Y, and let $U = \Sigma Y_i$. What are E(U), Var(U) ?

(a) $\frac{n\theta}{2}, \frac{n\theta^2}{12}$ (b) $n\theta, n\theta^2$ (c) $\frac{n\theta}{2}, \frac{\theta^2}{n}$ (d) $\frac{\theta}{2}, \frac{\theta^2}{12}$ (e) $\frac{\theta}{2}, \frac{\theta^2}{12n}$

7. Let Y be exponential with $f_Y(y) = \lambda e^{-\lambda y}$ for $y > 0$. Let $Y_1,...,Y_n$ be a random sample from Y. What is the maximum likelihood estimator for λ ?

(a) $\frac{\Sigma Y_i}{n}$ (b) $\frac{n}{\Sigma Y_i}$ (c) Y_1' (d) $\frac{1}{Y_n'}$ (e) $\frac{\Sigma Y_i^2}{n}$

8. Of 100 people questioned, 64 say that they would favor a tax increase to improve public schools. Give 95% confidence interval estimate for the over-all proportion.

(a) $.64 \pm (.024)(1.96)$ (b) $.64 \pm (.036)(1.96)$ (c) $.64 \pm (.048)(1.96)$
(d) $.64 \pm (.064)(1.96)$ (e) $.64 \pm (.072)(1.96)$

9. The weights of salmon caught in Lake Michigan once averaged 4 lbs. There is concern whether over-fishing or chemical contaminants may have reduced the number of large fish. The plan is to test
H_0: $\mu = 4$ v. H_1: $\mu < 4$, at the 5% level of significance, using a sample of size 25. Suppose the weights are normally distributed with $\sigma^2 = 1$. When would the natural decision rule say to reject H_0 ?

(a) $5(\tilde{X} - 4) \leq 1.96$ (b) $5(\tilde{X} - 4) \leq 1.71$ (c) $5(\tilde{X} - 4) \leq 1.65$

(d) $5(\tilde{X} - 4) \leq 2.06$ (e) $5(\tilde{X} - 4) \leq -1.65$

10. Suppose in Problem 9, σ^2 is unknown. When would the natural decision rule say to reject H_0 ?

(a) $\frac{5(\tilde{X} - 4)}{s} \leq 1.96$ (b) $\frac{5(\tilde{X} - 4)}{s} \leq 1.71$ (c) $\frac{5(\tilde{X} - 4)}{s} \leq 1.65$

(d) $\frac{5(\tilde{X} - 4)}{s} \leq 2.06$ (e) $\frac{5(\tilde{X} - 4)}{s} \leq -1.65$

11. Let X be normal with mean μ and variance σ^2 (unknown). Suppose you wish to test the hypotheses H_0: $\mu = 5$ v. H_1: $\mu \neq 5$, at the 5% level of significance, using a sample of size 16. When would the natural decision rule say to reject H_0 ?

(a) $\left| \frac{4(\tilde{X} - 5)}{s} \right| \geq 2.1315$ (b) $\left| \frac{4(\tilde{X} - 5)}{s} \right| \geq 1.7530$ (c) $\frac{4(\tilde{X} - 5)}{s} \geq 2.1315$

(d) $\frac{4(\tilde{X} - 5)}{s} \geq 1.7530$ (e) $\frac{4(\tilde{X} - 5)}{s} \geq 1.7459$

12. Suppose X is a random variable with mean μ and variance σ^2, and let $\tilde{X}_n$ be the mean of a random sample of size n from X. The Central Limit Theorem says that for an interval I,

$$\lim_{n \to \infty} P\left(\frac{\tilde{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}} \in I \right) = P(Z \in I) \text{ under what conditions ?}$$

(a) X is normal (b) X is binomial (c) X is continuous
(d) always (e) CLT says nothing of the sort

13. A sample of size 9 from normal random variable yields $s^2 = 16$. Find a 90% confidence interval estimate for σ^2.

(a) $\left(\frac{120}{15.507}, \frac{120}{2.733}\right)$ (b) $\left(\frac{128}{15.507}, \frac{128}{2.733}\right)$ (c) $\left(\frac{2.733}{120}, \frac{15.507}{120}\right)$

(d) $\left(\frac{2.733}{128}, \frac{15.507}{128}\right)$ (e) (2.733 , 15.507)

14. A sample of 30 students at Notre Dame and 30 at IU are asked whether they are worried about job prospects.

	worried	not worried
ND	10	20
IU	20	10

The plan is to test H_0: Worry independent of school v. H_1: Worry not independent on school, using the statistic $U = \sum_{i,j} \frac{(X_{i,j} - X'_{i,j})^2}{X'_{i,j}}$, where $X_{i,j}$ is the observed number in row i and column j, and $X'_{i,j}$ is the corresponding predicted number. Give a decision rule for a test at the 5% level of significance, and calculate the value of U.

(a) Reject H_0 iff $U \geq 9.488$, $U = \frac{20}{3}$ (b) Reject H_0 iff $U \geq 3.841$, $U = \frac{20}{3}$

(c) Reject H_0 iff $U \geq 9.488$, $U = \frac{10}{3}$ (d) Reject H_0 iff $U \geq 3.841$, $U = \frac{10}{3}$

(e) Reject H_0 iff $U \geq 7.815$, $U = \frac{10}{3}$

15. Suppose X and Y have joint pdf $f_{X,Y}(x,y) = 2$ for $x,y > 0$, $x+y < 1$. Find Cov(X,Y).

(a) $-\frac{1}{36}$ (b) $-\frac{1}{12}$ (c) 0 (d) $\frac{1}{12}$ (e) $\frac{1}{36}$

16. For X and Y as in Problem 17, find the regression line of Y on X.

(a) $E(Y|x) = 2 + x$ (b) $E(Y|x) = 1 + x$ (c) $E(Y|x) = .5 - .5x$
(d) $E(Y|x) = .5 + 2x$ (e) $E(Y|x) = 2 - .5x$