1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 50 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 5 PAGES with 10 PROBLEMS. The point value for each problem is 10 points with a total of 100 points. The blank sheet and the backs of the test pages are for scratch work.
3. On all problems, show your work, indicating clearly in the space provided how you arrived at your answer. The points you receive for problems 6 to 10 will depend on the extent to which the work you show convinces the grader (at the time of grading) that you did all or part of the problem correctly, whether or not you write down the correct answer.

Sign your nam

12 =2 =2.5in =0.8cm =1cm =0.4cm

=1

Find the general solution to

$$y'' - 4y' + 5y = 0$$

$y = e^{-3x}\left(c_1\cos(x) + c_2\sin(x)\right)$ $y = c_1\cos(\frac{3}{2}x) + c_2\sin(\frac{3}{2}x)$ $y = c_1e^{-3x} + c_2e^{3x}$ $y = c_1e^{+2x}\cos(x) + c_2e^{+2x}\sin(x)$ $y = c_1e^{-2x}\cos(x) + c_2xe^{-2x}\sin(x)$

Assume the characteristic equation of a sixth–order differential equation is $(r^3-1)(r-1)^2 = 0$. Which one of the following functions is a solution? (Only one is.)

$\cosh(x)$ $x\cos(\frac{\sqrt{3}}{2}x)$ $e^{x2}\sin x$ xe^{-x} x^2e^x

Assume $\mathcal{L}\{f(t)\} = \sqrt{\frac{\pi}{s-2}}$. Then $\mathcal{L}\{e^{-4t}f(t)\} =$

$\sqrt{\frac{\pi}{s-2}}$ $e^{-2s}\sqrt{\frac{\pi}{s}}$ $e^{-2s}\sqrt{\frac{\pi}{s-2}}$ $\frac{\sqrt{\pi}}{s-2}$ $\sqrt{\frac{\pi}{s+2}}$

Denote with $u_c(t)$ the unit step function. Find the Laplace transform of $f(t) = t + u_1(t)(t-1)$.

$F(s) = \frac{1-e^{-s}}{s^2}$ $F(s) = \frac{1+e^{-s}}{s^2}$ $F(s) = \frac{1}{s^2}$ $F(s) = \frac{2+3s}{s^2+1}$ $F(s) = s^{-2} + e^{-2s}$ Denote with $\delta(t)$ the Dirac delta function. Calculate the Laplace Transform of:

$$f(t) = tSin\frac{\pi}{2}t\delta(t-1)$$

$\frac{2}{s^2}$ $\frac{\delta(2)}{s}$ $\frac{1}{\int s}$ $\frac{e^{-s}}{s}$ e^{-s}

Solve the initial value problem

$$y^{(4)} = \delta(t-5); \qquad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0,\ y'''(0) = 1.$$

$\frac{1}{6}u_5(t)(t-5)^3$ $\frac{1}{6}u_5(t)(t-5)^3$ $u_2(t)(t-5)^3 + (t-5)^2$

$\frac{1}{2}u_5(t)(t-5)^2 + \frac{(t)^3}{6}$ $\frac{1}{3}u_6(t)(t-5)^3$

Solve the initial value problem:

$$y^{(4)} - 16y = 0; \qquad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1,\ y'''(0) = 0.$$

Calculate the Wronskian, i.e. the determinant of the Wronskian matrix, of the functions:

$$\{e^{ix}, \sin(x), \cos(x)\}.$$

Find the inverse Laplace transform of:

Consider the differential equation:

$$F(s) = \frac{e^{-s}}{s^2 + 2s - 3}$$

$$y'' - 2y' = 4e^{2x}\sin(x).$$

The solution of the homogenous equation is given by:

$$y = c_1 + c_2 x + c_3 e^{2x}.$$

a) Assume $y_p(x) = u_1(x) + u_2(x)x + u_3(x)e^{2x}$ is a particular solution. Use the method of "Variation of Parameters" to write down a system of equations satisfied by $u_1(x), u_2(x), u_3(x)$.

b)Solve for $u_3(x)$.