1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 50 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 5 PAGES with 10 PROBLEMS. The point value for each problem is 10 points with a total of 100 points. The blank sheet and the backs of the test pages are for scratch work.
3. On all problems, show your work, indicating clearly in the space provided how you arrived at your answer. The points you receive for problems 6 to 10 will depend on the extent to which the work you show convinces the grader (at the time of grading) that you did all or part of the problem correctly, whether or not you write down the correct answer.

Sign your nam

12 =0 =2.5in =0.8cm =1cm =0.4cm =1

Which picture best describes the trajectories of $x' = Ax$, $A = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$

a)

b)

c)

d)

e)

Write down a fundamental matrix for $\Phi(t)$ $x' = Ax$, $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, satisfying $\Phi(0) = I$.

a) Show that the vectors $x^{(1)} = \begin{pmatrix} te^t \\ t \end{pmatrix}$ and $x^{(2)} = \begin{pmatrix} e^t \\ 2 \end{pmatrix}$ form a linearly independent set on the interval $[-1, 1]$.

b) If these vectors satisfy a system $x' = Px$, what can be said about the elements of the matrix P?

a) Solve the system $x' = Ax$ where $A = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$.

b) Sketch the trajectories of the system.

Find two linearly independent real solutions of $x' = Ax$ where $A = \begin{pmatrix} -2 & -1 \\ 7 & 3 \end{pmatrix}$.

b) Sketch the trajectories.

Consider the system $x' = Ax$ where A has eigenvalues -3 and 1 with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

a) Sketch the trajectories of this system.

b) What is the type and stability of the equilibrium point $(0, 0)$?

c) Find the fundamental matrix $\Phi(t)$ for the system such that $\Phi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The general solution of the homogeneous system $x' = \begin{pmatrix} 4 & -3 \\ 8 & -6 \end{pmatrix} x$ can be shown to be

$x = c_1 \begin{pmatrix} 3 \\ 4 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$.

a) Using this information and the method of variation of parameters for the non-homogeneous system $x' = \begin{pmatrix} 4 & 3 \\ 8 & 0 \end{pmatrix} x + \begin{pmatrix} e^t \\ 2e^t \end{pmatrix}$, find the equations satisfied by the components u_1' and u_2' of the vector u'.

b) Find a particular solution of this non-homogeneous system.

Find the eigenvalues and eigenvectors for the matrix $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{pmatrix}$

Solve the initial value problem $x' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} x$, $x(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$x(t) = \begin{pmatrix} 3 + 4t \\ 2 + 4t \end{pmatrix} e^{3t}$

$x(t) = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} t \\ t \end{pmatrix} e^{-3t}$

$x(t) = \begin{pmatrix} 3 \\ 2t \end{pmatrix} + \begin{pmatrix} 1 \\ 1t \end{pmatrix} e{-}3t$

$x(t) = \begin{pmatrix} 3-t \\ 2+t \end{pmatrix} e^{3t}$

$x(t) = \begin{pmatrix} 1 \\ 4t \end{pmatrix} e^{3t} + \begin{pmatrix} -4 \\ -7 \end{pmatrix} e{-}3t$

Which of the following numbers is an eigen value of

$$A = \begin{pmatrix} 7 & 2 & 2 & 2 \\ 2 & 7 & 2 & 2 \\ 2 & 2 & 7 & 2 \\ 2 & 2 & 2 & 7 \end{pmatrix}$$

Hint: No calculation needed.

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$1+i$

$1-i$

2

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Consider the system $x'(t) = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x$. Find the fundamental matrix $\Phi(t)$ satisfying $\Phi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Phi(t) = \begin{pmatrix} \cos t + 2\sin t & -5\sin t \\ \sin t & \cos t - 2\sin t \end{pmatrix}$

$\Phi(t) = \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$

$\Phi(t) = \begin{pmatrix} 1+t & t^2 \\ t^2 - 2t & 1 \end{pmatrix}$

$\Phi(t) = \begin{pmatrix} e^t & e^t + t - 1 \\ e^{-t} - 1 & e^{-t} \end{pmatrix}$

$\Phi(t) = \begin{pmatrix} 1 & t \\ -t^2 & e^t \end{pmatrix}$

Consider the system $x' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} e^{-2t} \\ -2e^t \end{pmatrix}$.

Find a particular solution:

$\begin{pmatrix} 12e^t \\ e^{-2t} \end{pmatrix}$

$\begin{pmatrix} \cosh t \\ 2\sinh t \end{pmatrix}$

$\begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{3t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

$\begin{pmatrix} 5 \\ 5 \end{pmatrix} e^{2t}$

$\begin{pmatrix} \cos t \\ -2\sin t \end{pmatrix}$

Calculate the asymptotic behavior of $x' = \begin{pmatrix} 2 & -3 \\ -4 & 6 \end{pmatrix} x$

Consider the system $x^0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x$.

a) Find the fundamental matrix $\Psi(t)$ satisfying $\Phi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

b) Show that the system is unstable. Given that $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ is an eigenvector of $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$, find the corresponding eigenvalue.

b) Is this eigenvalue of multiplicity 1, 2 or 3?