1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 50 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 5 PAGES with 10 PROBLEMS. The point value for each problem is 10 points with a total of 100 points. The blank sheet and the backs of the test pages are for scratch work.
3. On all problems, show your work, indicating clearly in the space provided how you arrived at your answer. The points you receive for problems 6 to 10 will depend on the extent to which the work you show convinces the grader (at the time of grading) that you did all or part of the problem correctly, whether or not you write down the correct answer.

Sign your nam

12 =2.5in =0.8cm =1cm =0.4cm

=1 Suppose $f(x)$ satisfies the initial value problem $y' = y + 4x$; $y(0) = 3$. Use Euler's method with stepsize $h = 1$ to estimate $f(2)$.(2 iterations).

16 0 -7 2 4

Consider the following nonlinear system:

$$\begin{aligned} x' &= x^2 + 5x + 6 \\ y' &= x - y - 4 \end{aligned}$$

Which of the following points is a critical point?

$(-3, -7)$ $(-3, -2)$ $(-2, 0)$ $(-2, -5)$ $(-3, 5)$

Find the fundamental period of $f(x) = \tan \frac{1}{3}x$.

3π π 2 $\frac{\pi}{2}$ $\frac{1}{3}\pi$

Find a solution of the heat conduction problem:

$$u_{xx} = u_t \quad 0 < x < 1,\ t > 0$$

$$u(0, t) = 0, \quad u(1, t) = 0 \text{ for all } t > 0,$$

and $u(x, 0) = 3 \sin 2\pi x + 4 \sin 6\pi x$.

$3e^{-4\pi^2 t} \sin 2\pi x + 4e^{-36\pi^2 t} \sin 6\pi x$ $e^{-36\pi^2 t} \sin 2\pi x + 3e^{-4\pi^2 t} \sin 6\pi x$ $3e^{t} \sin 2\pi x + 4e^{t} \sin 6\pi x$ $3e^{-4\pi^2 t} \sin 2\pi x + 4e^{-6\pi^2 t} \sin 6\pi x$ $3e^{-2\pi t} \sin 2\pi x + 4e^{-6\pi t} \sin 6\pi x$

Find the coefficient of $\cos(2\pi x)$ in the Fourier Series of the function $f(x) = -2\sin(2\pi x) + \cos(3\pi x)$ defined on $[-1, 1]$. 0 π $\frac{\pi}{2}$ 5 -1

Suppose $f(x)$ is a function whose graph passes through the point $P = (0, 1)$ in the xy-plane and satisfies the differential equation $y' = (1 + y^3) \cos x$. Which of the following lines is the tangent line of the graph of f at P?

$y = 2x + 1$ $x + 1$ $2x - 1$ $x - 1$ $3x + 1$

Use the improved Euler method with stepsize $h = 0.1$ to estimate $y(1.1)$ for the initial value problem $y' = (\sqrt{x + y}\ )$, $y(1) = 3$. (You may approximate $\sqrt{4.3}$ by 2.08.)

If $u(x,t)$ =F(x)G(t) satisfies the partial differential equation

$$tu_{xx} - xu_t = 0$$

Then use the method of separation of variables to replace this partial differential equation by an ordinary differential equation for F and an ordinary differential equation for G.

Consider the non-linear system:

$$\begin{aligned} x' &= x^2 - y^2 \\ y' &= 1 - x \end{aligned}$$

a. Show that the point $(1,1)$ is a critical point for this system.

b. Determine the type and stability for this equilibrium point.

c. Find all other critical points for this system.

Let $f(x) = x$ for $-1 \leq x < 1$ and $f(x+2) = f(x)$, so that f is periodic with period 2. If we represent f by the Fourier series

$$\frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos(m\pi x) + b_m \sin(m\pi x))$$

determine the values of a_0, a_1, and b_1.