1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 120 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 9 PAGES with 15 PROBLEMS. The point value for each problem is 10 points with a total of 150 points. The blank sheet and the backs of the test pages are for scratch work.
3. **On all partial credit problems, show your work, indicating clearly how you arrived at your answer. The points you receive for problems 12 to 15 will depend on the extent to which the work you show convinces the grader (at the time of grading) that you did all or part of the problem correctly.**

Sign your name:

8 =2.5in =0.8cm =1cm =0.4cm

=1 Denote by $\delta(t)$ the Dirac delta function. Calculate the following improper integral:

$$\int_0^\infty te^{t^2-9}\delta(t-3)dt$$

3 $\delta(3)$ e^{23} 0 e

Solve the initial value problem

$$y''+y=\delta(t-\pi)-\delta(t-2\pi); \qquad y(0)=0,\ y'(0)=0.$$

$u_\pi(t)\sin(t-\pi)-u_{2\pi}(t)\sin(t-2\pi)$ $\frac{1}{3}u_\pi(t)\sin(t-\pi)-\frac{2}{3}u_{2\pi}(t)\sin(t-2\pi)$ $u_2(t)(t-5)+u_\pi(t)\cos(t-\pi)-u_{2\pi}(t)\sin(t-2\pi)$ $\frac{1}{2}u_\pi(t)\cos(t-\pi)-u_{2\pi}(t)\sin(t-2\pi)$ $\frac{1}{3}u_\pi(t)\sin(t-\pi)-u_{2\pi}(t)\sin(t-2\pi)$

Let $y(x)=u_1(x)\cos(x)+u_2(x)\sin(x)$ be the solution of

$$y''+y=\sin(x)\cos(x)$$

given by the method of variation of parameters. Find u_1. $u_1=-\frac{1}{3}\sin^3(x)$ $u_1=-\cot(x)$ $u_1=-\sin^2(x)\tan(x)$ $u_1=\tan(x)$ $u_1=\frac{1}{2}\cos^2(x)$

If $\mathcal{L}\{f(t)\}=\frac{1}{\sqrt{s^2+9}}$ and $f(0)=7$. Then what is $\mathcal{L}\{f'(t)\}$

$\frac{1}{\sqrt{s^2+9}}-7$ $\frac{1}{\sqrt{s^2+9}}+7$ $s\sqrt{s^2+9}+7$ $\sqrt{\frac{1}{s-7}}$ $\sqrt{\frac{7}{s+2}}$

Solve the initial value problem $x'=\begin{pmatrix}1 & -4\\ 4 & -7\end{pmatrix}x$, $x(0)=\begin{pmatrix}3\\2\end{pmatrix}$.

$$x(t)=\begin{pmatrix}3+4t\\2+4t\end{pmatrix}e^{-3t}$$

$$x(t)=\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}t\\t\end{pmatrix}e^{-3t}$$

$$x(t)=\begin{pmatrix}3\\2t\end{pmatrix}+\begin{pmatrix}1\\1t\end{pmatrix}e^{-3t}$$

$$x(t)=\begin{pmatrix}3-t\\2+t\end{pmatrix}e^{3t}$$

$$x(t)=\begin{pmatrix}1\\4t\end{pmatrix}e^{3t}+\begin{pmatrix}-4\\-7\end{pmatrix}e^{3t}$$

Consider the system $x'=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}x$. The fundamental matrix $\Phi(t)$ satisfying $\Phi(0)=I$ is:

$\frac{1}{2}\begin{pmatrix}e^{2t}+1 & e^{2t}-1\\ e^{2t}-1 & e^{2t}+1\end{pmatrix}$ $\begin{pmatrix}1+t & e^t\\ e^{2t} & e^t\end{pmatrix}$ $\frac{1}{3}\begin{pmatrix}2e^{2t}+1 & e^{2t}-1\\ e^{2t}-1 & e^{2t}+2\end{pmatrix}$ $\frac{1}{2}\begin{pmatrix}e^{2t}+1 & 2t^2-t\\ e^{2t}-1 & 2t^2-t\end{pmatrix}$ $\begin{pmatrix}1 & e^{2t}-1\\ e^{2t}-1 & 1\end{pmatrix}$

Let f and g be functions defined on $[-\pi,\pi]$ with f odd and g even. Then The Fourier Series of fg consists only of sine terms. The Fourier Series of fg consists only of cosine terms. The Fourier Series of fg^2 consists only of cosine terms. The Fourier Series of f^2g consists only of sine terms. The Fourier Series of f^2g^2 consists only of sine terms.

Suppose $f(x)$ satisfies the differential equation $y'=y-4$; $y(0)=5$. Use Euler's method with stepsize $h=1$ to estimate $f(3)$. (3 iterations).

12 0 5 14 4

Consider the function $f(x)=-x; -1\le x<1; f(x+2)=f(x)$. Find the Fourier Series expansion of $f(x)$.

$f(x)=\frac{2}{\pi}\sum_{n=1}^\infty\frac{(-1)^n}{n}\sin n\pi x$ $f(x)=\sum_{n=1}^\infty\frac{2}{n\pi}\sin n\pi x$. $f(x)=\sum_{n=1}^\infty\frac{(-1)^n}{n\pi}\cos n\pi x$. $f(x)=\frac{3}{\pi}\sum_{n=1}^\infty\frac{(-1)^{n+1}}{n}\cos n\pi x$. $f(x)=-x\cdot\sin n\pi x$.

Calculate the Wronskian, i.e. the determinant of the Wronskian matrix, of the functions:

$$\{\sin(x),e^{-ix},\cos(x)\}.$$

0 πix $\frac{2}{3}\pi x$ $2e^x$ e^{2ix}

If $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is an eigenvector of the symmetric matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ then which of the following vectors is an eigenvector as well?

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} \begin{pmatrix} -3 \\ 2 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Consider the non-linear system:

$$\begin{aligned} x' &= x + y \\ y' &= \sin(x - y) \end{aligned}$$

a. Show that the origin $(0, 0)$ is a critical point for this system.

b. Determine the type and stability for this equilibrium point.

c. Find all other critical points for this system.

Find the inverse Laplace transform of:

$$F(s) = \frac{s - 2}{s^2 - 4s + 13}$$

Find the condition satisfied by the positive real eigenvalues λ of the following boundary value problem:

$$y'' + \lambda y = 0 \qquad y(0) = 0 \quad y(\pi) + y'(\pi) = 0.$$

A metal wire 100cm long has a uniform temperature of $30°C$. At time $t = 0$ (today, Thursday Dec. 19 at 15:45pm E.S.T.) one end is held at a constant temperature of $10°C$ and the other end is held at a constant temperature of $70°C$. If no heat is lost along the wire then:

a) What will be the approximate temperature distribution $v(x)$, $0 \le x \le 100$ on Christmas eve?

b) Using $\alpha^2 = 4$ in the heat equation find the temperature distribution $u(x, t)$. (Write down $u(x, t)$ as a series and then express the coefficients in this series as integrals.)

c) (5 points Christmas bonus!) Evaluate these coefficients and get a series expansion of $u(x, t)$

valid for $0 \leq x \leq 100$, $t > 0$.

$Merry\ Christmas$!!!