1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 50 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 7 PAGES with 11 PROBLEMS. The point value for each multiple choice problem is 8 points. Problems 9, 10 and 11 are worth12 points each giving a total of 100 points. Use the back of the test pages for scratch work.
3. On all partial credit problems, show your work, indicating clearly how you arrived at your answer. The points you receive for problems 9, 10 and 11 will depend on the extent to which the work you show convinces the grader (at the time of grading) that you did all or part of the problem correctly.
4. At the end of the test the table of Laplace transforms is attached.

Sign your name:

12 =3 =2.5in =0.8cm =1cm =0.4cm

=1

Find the general solution of the differential equation

$$y'' + 10y' + 26y = 0$$

$y = e^{-5x}(c_1\cos(x) + c_2\sin(x))$ $y = c_1\cos(\frac{3}{2}x) + c_2\sin(\frac{3}{2}x)$ $y = c_1e^{-5x} + c_2e^{5x}$ $y = e^{10x}(c_1\cos(x) + c_2\sin(x))$ $y = c_1e^{-4x}\cos(x) + c_2xe^{-4x}\sin(x)$

Assume $\mathcal{L}\{f(t)\} = \cos(s)$. Then $\mathcal{L}^{-1}\{\cos(s+2)\} =$

$e^{-2t}f(t)$ $e^{2t}f(t)$ $f(t-2)$ $e^{3t}f(t-3)$ $f(t+3)$

Denote with $u_c(t)$ the unit step function. Find the Laplace transform of $f(t) = u_3(t)(t-5)$.

$F(s) = \frac{e^{-3s} - 2se^{-3s}}{s^2}$ $F(s) = \frac{1+e^{-3s}}{s}$ $F(s) = \frac{1}{s^3}$ $F(s) = \frac{e^{-3s}}{s^2+1}$ $F(s) = s^{-2} + e^{-3s}$

Calculate the Wronskian determinant of the functions:

$$\{\cosh(x), e^x, e^{-x}\}.$$

0 e^{3x} $\cosh(x)\sinh(x)$ 1 $\cosh(x)$

Assume the characteristic equation of a sixth-order differential equation is

$$(r^2+1)^2(r-1)(r+2) = 0.$$

Which one of the following functions is a solution? (Only one is.)

$x\cos(x)$ xe^{-2x} $x\cos(2x)$ $x^2\sin(x)$ x^2e^x

Let $y(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ be a particular solution of

$$y'' + y = \tan(x), \qquad 0 < x < \frac{\pi}{2}$$

obtained by the method of variation of parameters. Find u_2.

$u_2 = -\cos(x)$ $u_2 = \sin(x)$ $u_2 = \sec^2(x)$ $u_2 = -\sec(x)$ $u_2 = \cot(x)$

Find the inverse Laplace transform of the function

$$F(s) = \frac{2e^{-4s}}{s^2+4} \quad .$$

$u_4(t)\sin 2(t-4)$ $u_4(t)e^{-2(t-4)}$ $u_4(t)\cos 2(t-4)$ $u_4(t)(t-4)$ $u_4(t)e^{-4(t-2)}$

Solve the initial value problem

$$y'' + 2y' + y = 0; \qquad y(0) = -3,\ y'(0) = 4.$$

$y = e^{-x}(x-3)$ $y = (-3)e^x + xe^x$ $y = (-3)e^x + xe^{-x}$ $y = (7x-3)e^{-x}$ $y = (7x-3)e^x$

(12 pts.) Use the Laplace transform to solve the initial value problem:

$$y'' + 4y' + 4y = te^{-2t}, \quad y(0) = 0,\ y'(0) = 1.$$

(12 pts.) Use the method of undetermined coefficients to solve the initial value problem:

$$y''+10y'+25y = 108xe^{x}+25 \qquad y(0) = 0,\ y'(0) = 0.$$

(12 pts.) Solve the initial value problem

$$y''+4y = g(t); \qquad y(0) = 0,\ y'(0) = 0, \quad g(t) := \begin{cases} 0 & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ 0 & 2 \le t < \infty \end{cases}$$