1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 50 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 6 PAGES with 10 PROBLEMS. The point value for each multiple choice problem is 9 points. Problems 9 and 10 are worth 14 points each giving a total of 100 points. Use the back of the test pages for scratch work.
3. On all partial credit problems, show your work, indicating clearly how you arrived at your answer. The points you receive for problems 9 and 10 will depend on the extent to which the work you show convinces the grader (at the time of grading) that you did all or part of the problem correctly.

Sign your name:

6 =3 =2.5in =0.8cm =1cm =0.4cm

=1 The inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 5 & 3 \end{pmatrix}$ is:

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -5 & 2 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 5 \\ 0 & 1 & -3 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 5 \\ 0 & 1 & -3 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -5 & 2 \end{pmatrix}$ $\begin{pmatrix} 2 & 1 & 0 \\ 5 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Consider the following system of 3 equations in the 3 unknowns x_1, x_2, x_3.

$$\begin{array}{ccccccc} x_1 & + & x_2 & + & x_3 & = & a \\ x_1 & + & 2x_2 & + & 3x_3 & = & a \\ x_1 & + & 7x_2 & + & 13x_3 & = & a \end{array}$$

Which of the following statements is true?

One has infinitely many solutions x for any given value a. One has no solution x if $a = 0$. One has no solution $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ if $a \neq 0$. One has only one solution x if $a = 0$. One has exactly one solution x if $a \neq 0$.

Which of the following numbers is an eigenvalue of

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

0 1 2 4 8

Find the general solution of $x' = Ax$ where $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

$c_1 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} -\cos t \\ \sin t \end{pmatrix}$

$c_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$c_1 \begin{pmatrix} \cos t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \sin t \end{pmatrix}$

$c_1 \begin{pmatrix} e^t \cos t \\ -e^t \sin t \end{pmatrix} + c_2 \begin{pmatrix} e^t \cos t \\ e^t \sin t \end{pmatrix}$

$c_1 e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

You are given a matrix $A = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}$ which has a double eigenvalue at $r = 4$ with a corresponding eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Therefore the system $x' = Ax$ has the solution $\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t}$. Find the general solution for this system.

$x(t) = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} 2t - 1 \\ t \end{pmatrix} e^{4t}$

$x(t) = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + C_2 t \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t}$

$x(t) = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} 2t \\ t - 4 \end{pmatrix} e^{4t}$

$x(t) = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + C_2 t \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$

$x(t) = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} 1 \\ t \end{pmatrix} e^{4t}$

If $x(t)$ is the solution of the initial value problem

$$x' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix},$$

then $x(\pi 2)$ is:

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Find the unique solution of the initial value problem $x' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} e^{2t} \\ e^{2t} \end{pmatrix} \quad x(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

$\begin{pmatrix} e^{2t} + e^{-t} \\ e^{2t} - e^{-t} \end{pmatrix}$

$\begin{pmatrix} 2\cosh t \\ \sinh t \end{pmatrix}$

$\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

$\begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{2t}$

$\begin{pmatrix} 2\cos t \\ 2\sin t \end{pmatrix}$ Consider the system $x' = Ax$, where A has eigenvalues -3 and 1 with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. If $\Phi(t)$ is the fundamental matrix with $\Phi(0) = I$ then the determinant of $\Phi(t)$ equals:

e^{-2t} e^{t} e^{-3t} e^{2t} e^{3t} (14 pts.) Find the general solution of the system $x' = \begin{pmatrix} 3 & 5 & 7 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} x$.

(14 pts.) Consider the system $x' = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x$. Find the fundamental matrix $\Phi(t)$ satisfying $\Phi(0) =$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$