READ THESE DIRECTIONS

1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 50 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 5 PAGES with 7 PROBLEMS. The point value for each multiple choice problem is 12 points. Problem 7 is worth 24 points (with additional 4 points for free). Use the back of the test pages for scratch work.
3. On all partial credit problems, show your work, indicating clearly how you arrived at your answer. The points you receive for problem 7 will depend on the extent to which the work you show convinces the grader (at the time of grading) that you did all or part of the problem correctly.

Sign your name

12

=3

=2.5in =0.8cm =1cm =0.4cm

=1

Which of the sketches on the last page of this exam best describes the trajectories of $x' = Ax$,

$$A = \begin{pmatrix} 5 & 1 \\ 3 & 3 \end{pmatrix}$$

4 2 3 7 1

Which of the sketches on the last page of this exam best describes the trajectories of $x' = Ax$,

$$A = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$$

8 1 7 5 2

Consider the system

$$\begin{aligned} x' &= \tfrac{1}{y-1} \\ y' &= x^2 \end{aligned} .$$

Determine the trajectories of this system by finding $\frac{dy}{dx}$ and integrating.

$y = C\exp\left(\frac{x^3}{3}\right) + 1$ $y = C\exp\left(\frac{x^3}{3}\right) - 1$ $y = C\left(\frac{x^2}{2}\right) + 1$ $y = \exp\left(\frac{x^3}{2}\right) - 1$ $x^2 + y^2 = C$.

For the system

$$\begin{aligned} x' &= x + y \\ y' &= -x - xy \end{aligned}$$

the critical point $(1, -1)$ is:

a saddle point a center a spiral point. a proper node. an improper node.

Consider the system:

$$\begin{aligned} x' &= -x^3 \\ y' &= -y^3. \end{aligned}$$

Using a Liapunov function V(x,y) = $ax^2 + cy^2$ one can say that the critical point (0,0) is:

asymptotically stable. a saddle point. a spiral point. stable but not asymptotically stable. unstable.

(In the answers below n stands for all positive integers and zero.)

The critical points of the system

$$\begin{aligned} x' &= x \sin y \\ y' &= (x^2 - 1)(y - 1) \end{aligned}$$

are:

$(0, 1)$, $(1, \pm n\pi)$ and $(-1, \pm n\pi)$. (0,1) and (1,0). $(0, n\pi)$ and $(1, n\pi)$. $(n\pi, n\pi)$. $(0, 1), (1, 0)$ and $(n\pi, n\pi)$.

a) The equation

$$\frac{d^2\theta}{dt^2} + \sin\theta + \frac{d\theta}{dt} = 0$$

can be written as a system of two first order differential equations

$$\begin{aligned} x' &= y \\ y' &= -\sin x - y \end{aligned}$$

where $x = \theta$ and $y = \frac{d\theta}{dt}$.

a) List all critical points of this system.

b) Study the stability properties of the critical point $(0, 0)$.

c) Study the stability properties of the critical point $(\pi, 0)$.