MATH 325 Practice Test I
PART A (Multiple Choice)

1. (9 points) Find the general solution of the homogeneous equation

$$y^{v} - y^{iv} + 2y''' - 2y'' + y' - y = 0 \ .$$

using the fact that the characteristic equation is $(r^2 + 1)^2(r - 1) = 0$.

a. $c_1 e^x + c_2 \cos x + c_3 \sin x + c_4 x \cos x + c_5 x \sin x$
b. $c_1 e^x + c_2 x e^x + c_3 x^2 e^x + c_4 e^{-x} + c_5 x e^{-x}$
c. $c_1 e^x + c_2 \cos x + c_3 \sin x$
d. $c_1 e^x + c_2 e^{-x} + c_3 e^{2x} + c_4 e^{-2x} + c_5 e^{3x}$
e. $c_1 e^x + c_2 x e^x + c_3 x^2 e^x + c_4 \cos x + c_5 \sin x$

2. (9 points) Use the method of undetermined coefficients to find the general solution of the equation.

$$y'' + y = 2(x+2)e^x \ .$$

a. $c_1 \cos x + c_2 \sin x + (x + 1)e^x$
b. $c_1 e^x + c_2 e^{-x} + xe^x$
c. $c_1 \cos x + c_2 \sin x + xe^x$
d. $c_1 e^x + c_2 e^{-x} + (x - 3) e^x$
e. $c_1 e^x + c_2 e^{2x} + (x + 2) e^x$

3. (9 points) Let f(t) be the function defined by $f(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$.

From the definition of the Laplace transform, or otherwise, find the Laplace transform F(s) of f(t).

a. $\dfrac{1 - e^{-s}}{s}$
b. $\dfrac{e^{-s} + 1}{s}$
c. $\dfrac{e^{s} - 1}{s}$
d. $\dfrac{1 + e^{s}}{s}$
e. $\dfrac{e^{s} - e^{-s}}{s}$

4. (9 points) Find the inverse Laplace transform of the function $F(s) = \frac{1 + e^{-2s}}{s^2}$.

a. $t + u_2(t)(t - 2)$.

b. $u_2(t - 2)\ t$

c. $t + e^{t-2}$

d. $\sin t + u_2(t)$

e. $u_2(t)(t - 2) + t^2$

5. (9 points) If $\delta(t)$ denotes the Dirac delta function then the improper integral

$$\int_0^{\infty} e^{-(t-\frac{\pi}{2})} \sqrt{5 - \sin t}\ \ \delta\left(t - \frac{\pi}{2}\right) dt$$

converges to:

a. 2 b. $\sqrt{5}$ c. 0

d. $\frac{\pi}{2}$ e. 1

6. (8 points) Find the Laplace transform of

$$f(t) = \delta\left(t - \frac{\pi}{2}\right) \sin t.$$

a. $e^{\frac{-\pi s}{2}}$

b. $\delta\left(t - \frac{\pi}{2}\right) \frac{1}{1 + s^2}$

c. $e^{\frac{\pi s}{2}}$

d. $1 + e^{\frac{-\pi s}{2}}$

e. $e^{\frac{-\pi s}{2}} - 1$

7. (9 points) Find the solution of the initial value problem:

$$y'' + 4y = \sin t$$

$$y(0) = 0,\ y'(0) = 0$$

a. $\frac{1}{3}\sin t - \frac{1}{6}\sin 2t$

b. $-\frac{1}{2}\sin 2t + \sin t$

c. $\frac{1}{3}\sin t + \frac{1}{6}\sin 2t$

d. $\frac{1}{6}\sin t + \frac{1}{3}\sin 2t$

e. $\frac{1}{2}\sin 2t - \frac{1}{3}\sin t.$

8. (9 points) Find the Wronskian determinant of the three functions

$$\{x, x^2, \frac{1}{x}\}$$

a. $\frac{6}{x}$

b. $6/x^2$

c. $2/x^3$

d. $-4/x^2$

e. $-\frac{4}{x}$

PART B (Partial Credit)

9. (14 points) If $L[y] = y''' + by'' + cy' + dy = 0$ has $\{1, x, e^x\}$ as a fundamental set of solutions then use the method of variation of parameters to find a particular solution of

$$L[y] = e^x\ .$$

The solution should be written in the form $u_1 + u_2\, x + u_3\, e^x$ and all integration must be carried out for full credit.

10. (14 points) Find the solution of the initial value problem

$$y'' + 2y' + y = t\,\delta(t - \pi)$$
$$y(o) = 0,\ y'(o) = 0\ .$$