Math 325
Test 2

Part A (Multiple Choice, 8 points each)

1. Compute the Wronskian of the vectors

$$x^{(1)} = \begin{pmatrix} 2t \\ t \end{pmatrix} \text{ and } x^{(2)} = \begin{pmatrix} t^2 \\ 2 \end{pmatrix}$$

a. $4t - t^3$ b. $2t^3 + 2t$ c. 0 d. $t - 2$ e. $4t + t^3$

2. Find all of the eigenvalues of the matrix

$$\begin{pmatrix} 2 & 0 & 7 \\ 1 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

a. 2, 3, –1 b. 2, 3, 1 c. 0, 2, 3 d. –2, 3, 1 e. 2, –3, 1

3. The matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ has $\lambda = 1$ as a repeated eigenvalue of multiplicity 2.

Find all of the eigenvectors of A for this eigenvalue.

a. all non-zero multiples of $\begin{pmatrix}1\\0\\0\end{pmatrix}$

b. all non-zero multiples of $\begin{pmatrix}0\\1\\0\end{pmatrix}$

c. all non-zero multiples of $\begin{pmatrix}0\\0\\1\end{pmatrix}$

d. all non-zero linear combinations of $\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\\1\end{pmatrix}$

e. all non-zero linear combinations of $\begin{pmatrix}0\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\1\end{pmatrix}$

4. Find the eigenvalues λ and the corresponding eigenvectors ξ for the matrix $\begin{pmatrix}2 & 0\\5 & 0\end{pmatrix}$

a. $\lambda = 0, \xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$; $\lambda = 2, \xi = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

b. $\lambda = 0, \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\lambda = 2, \xi = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

c. $\lambda = 0, \xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$; $\lambda = -2, \xi = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

d. $\lambda = 1, \xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\lambda = 2, \xi = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

e. $\lambda = 0, \xi = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$; $\lambda = 5, \xi = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

5. One of the following is a real solution of the equation $x' = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} x$.

a. $\begin{pmatrix} \cos 2t \\ -\sin 2t \end{pmatrix}$ b. $\begin{pmatrix} \sin 2t \\ 2\cos 2t \end{pmatrix}$ c. $\begin{pmatrix} \cos 2t \\ \sin 2t - \cos 2t \end{pmatrix}$

d. $\begin{pmatrix} \sin 2t \\ \sin 2t + \cos 2t \end{pmatrix}$ e. $\begin{pmatrix} \sin 2t + \cos 2t \\ \sin 2t - \cos 2t \end{pmatrix}$

6. Find an eigenvector with eigenvalue $\lambda = 2$ for the matrix $\begin{pmatrix} 2 & 5 & 0 & 8 \\ 3 & 6 & 0 & 2 \\ 4 & 2 & 2 & 4 \\ 1 & 0 & 0 & 7 \end{pmatrix}$.

a. $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ b. $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ c. $\begin{pmatrix} 1 \\ -1 \\ 2 \\ 3 \end{pmatrix}$ d. $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 7 \end{pmatrix}$ e. $\begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$

PART B (Partial credit)

7.(10 points) Solve the initial value problem

$$y'' + y = 1 - u_\pi(t)$$

$$y(0) = 0,\ y'(0) = 0$$

by use of convolution.

8.(10 points) The set of vectors $x^{(1)} = (-2, 2, 2)$, $x^{(2)} = (-1, 2, 0)$, $x^{(3)} = (-1, -2, 4)$ is linearly dependent. Find numbers c_1, c_2, c_3 such that $c_1 x^{(1)} + c_2 x^{(2)} + c_3 x^{(3)} = 0$.

9.(4 points) a. Find the eigenvalues and eigenvectors for the matrix $A = \begin{pmatrix} 5 & 3 \\ 0 & 5 \end{pmatrix}$.

(8 points) b. Use the information from part a of this problem to find the general solution of equation $x' = \begin{pmatrix} 5 & 3 \\ 0 & 5 \end{pmatrix} x$.

10.(10 points) Find a fundamental set of solutions of the equation

$$\frac{dx}{dt} = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x .$$

11.(10 points) Find a matrix T satisfying the equation

$$T^{-1} \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$