PRACTICE TEST 3 MATH 325 November 30, 1994

Multiple Choice . (9 points each)

In problems 1 and 2 refer to the sketches of trajectories on page 2 of the exam. Note that directions of eigenvectors are only approximate.

1. The trajectories of the system $x' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x$ are of the type

a. (2) b. (1) c. (8) d. (4) e. (5)

2. The trajectories of the system $x' = \begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix} x$ are of the type

a. (2) b. (6) c. (8) d. (7) e. (3)

3. What is the type and stability of the point (0, 0) for the system $x' = \begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix} x$?

a. center, stable
b. improper node, stable
c. improper node, unstable
d. spiral, unstable
e. saddle point, unstable

4. The trajectory of the system $x' = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} x$ passing through the point (0, 1) is tangent to the vector

a. $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ b. $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ c. $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ d. $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ e. $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

5. Find all of the critical points of the system

$$x' = (x - 1)\, y$$
$$y' = x\,(y^3 + 1)$$

a. only (0,0) b. (0,0) and (1,–1) c. (0,0), (1,–1) and (1.1)
d. (0,0) and (1,0) e. (0,0) and (0,1)

6. Let ϕ (t) be the solution to the intial value problem

$$\begin{cases} y' = y^2 + t \\ y(0) = 0 \end{cases}$$

What is the approximate value of $\phi(2)$ using the Euler method with stepsize h = 1.0?

a. 0.2 b. 0.5 c. 1. d. 2 e. 3

7. If the method of separation of variables is used on the equation

$2u_{xx} - 3u_t = 0$ by setting $u(x,t) = X(x)T(t)$, the resulting pair of ordinary differential equations are

a. $\frac{3X''}{T} = \frac{2T'}{X}$ = constant

b. $\frac{2X''}{3X} = \frac{T''}{T}$ = constant

c. $\frac{X''}{X} = \frac{T'}{T}$ = constant

d. $\frac{2X''}{3X} = \frac{T'}{T}$ = constant

e. $\frac{3X''}{2X} = \frac{2T''}{3T}$ = constant

PARTIAL CREDIT

8. Consider the nonhomogeneous system

$$x' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x + \begin{pmatrix} 5e^{-2t} \\ 0 \end{pmatrix}$$

We want to solve it using the method of variation of parameters. A fundamental system of solutions to the homogeneous system is $x^{(1)} = \begin{pmatrix} e^{-3t} \\ -4\ e^{-3t} \end{pmatrix}$, $\qquad x^{(2)} = \begin{pmatrix} e^{2t} \\ e^{2t} \end{pmatrix}$

Let $x\ (t) = u_1\ x^{(1)} + u_2\ x^{(2)}$ be a solution.

a. (5 points) Write equations satisfied by u'_1 and u'_2.

b. (4 points) Find u_1 and u_2 .

c. (5 points) Find the general solution of the given nonhomogeneous system.

9. Consider the autonomous system

$$x' = F\ (x, y) = -\sin x + \cos y + y - 1$$
$$y' = G\ (x, y) = x - \pi + y^2$$

a. (4 points) Show that $x = \pi$, $y = 0$ is a critical point.

b. (5 points) What is the corresponding linear system near $(\pi, 0)$?

c. (5 points) What is the type of the critical point $(\pi, 0)$ of the given system?

10. (9 points) Suppose that $\phi\ (t)$ is the solution to the initial value problem

$$\begin{cases} y' = y + t + 1 \\ y\ (0) = 1 \end{cases}$$

Find an approximate value of $\phi\ (1.0)$ using the improved Euler method with stepsize $h = 1.0$.