Part I. Multiple choice 9 points each

1. The Laplace transform of $f(t) = te^t$ is

a. $F(s) = s + e^{-s}$

b. $F(s) = \dfrac{1}{(s-1)^2}$

c. $F(s) = \dfrac{1-s}{s^2}$

d. $F(s) = \dfrac{1}{s^2}$

e. $F(s) = s^{-2} + e^{-s}$

2. Assume $\mathcal{L}\{f(t)\} = \sqrt{\dfrac{\pi}{s-2}}$. Then $\mathcal{L}\{\, e^{-4t} f(t)\} =$

a. $\sqrt{\dfrac{\pi}{s-2}}$

b. $e^{-2s}\sqrt{\dfrac{\pi}{s}}$

c. $e^{-2s}\sqrt{\dfrac{\pi}{s+2}}$

d. $\sqrt{\dfrac{\pi}{s+2}}$

e. $\dfrac{\sqrt{\pi}}{s-2}$

3. Denote with $u_c(t)$ the unit step function. Find the Laplace transform of

$f(t) = t + u_1(t)(t-1)$.

a. $F(s) = s^{-2} + e^{-2\,s}$

b. $F(s) = \dfrac{1}{s^2}$

c. $F(s) = \dfrac{2+3s}{s^2+1}$

d. $F(s) = \dfrac{1-e^{-s}}{s^2}$

e. $F(s) = \dfrac{1+e^{-s}}{s^2}$

4. Denote by $\delta(t)$ the Dirac delta function. Calculate the following improper integral:

$$\int_0^{\bullet} e^{-\frac{t \cos t}{\pi}} \delta(t - \pi)\, dt$$

a. $\delta(2)$ b. 2 c. 0 d. The integral is undefined. e. e

5. Let $f(t) = \int_0^t \sin(t-\tau) \cos\tau \, d\tau$ The Laplace transform of $f(t)$ is

a. $\frac{1}{s^2+1} + \frac{s}{s^2+1}$ b. $\frac{s^2}{(s^2+1)^2}$ c. $\frac{s}{(s+1)^2}$

d. $\frac{s}{(s^2+1)^2}$ e. $e^{-s} \frac{s}{s^2+1}$

6. The solution of the initial value problem
$y'' + 4y = 2\,\delta(t - \frac{\pi}{4})$, $y(0) = 0$, $y'(0) = 0$ is

a. $u_{\frac{\pi}{4}}(t) \sin t$ b. $u_{\frac{\pi}{4}}(t) \sin(t - \frac{\pi}{4})$ c. $u_{\frac{\pi}{4}}(t) \cos 2(t - \frac{\pi}{4})$

d. $u_{\frac{\pi}{4}}(t) \sin(t - \frac{\pi}{4})$ e. $e^{-\frac{\pi}{4}t} \sin 2(t - \frac{\pi}{4})$

7. The eigenvalues of the matrix $\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$ are

a. 3 and 1 b. -3 and -1 c. 3 and -1 d. -3 and 1

e. 3 is a repeated root.

8. The matrix $\begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$ has an eigenvector $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$.

The eigenvalue which corresponds to this eigenvector is

a. 3 b. 2 c. 1 d. -1 e. -2

Part II. Partial Credit will be given for the following questions only if work is clearly shown. No credit for answer alone or unmotivated.

9.(10 points) Determine whether the vectors

$x^{(1)} = (1, 2, 0, -1)$, $x^{(2)} = (7, 4, 2, -3)$ and $x^{(3)} = (2, -1, 1, 0)$ form an independent or a dependent set.

10. (18 points) Let $A = \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix}$.

a. Find the eigenvalues and eigenvectors of A.

b. Solve the initial value problem $x' = Ax \quad x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

c. Sketch the trajectories of this system