I (8 points)

Which of the sketches on the last page best describe the trajectories of the system $x' = Ax$. Also determine stability of the system if:

1. $A = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$

Type____________________ Stability_________________ _

2. $A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

Type____________________ Stability_________________ _

3. $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

Type____________________ Stability_________________ _

4. The equation $\frac{d^2 y}{dt^2} - \frac{3dy}{dt} - e^t y = 0$ is equivalent to the first order system

(a) $\frac{dx_1}{dt} = x_1$

$\frac{dx_2}{dt} = 3x_1 + e^t x_2$

(b) $\frac{dx_1}{dt} = x_2$

$\frac{dx_2}{dt} = 3x_1 + e^t x_2$

(c) $\frac{dx_1}{dt} = x_1 - x_2$

$\frac{dx_2}{dt} = x_1 + x_2$

(d) $\frac{dx_1}{dt} = x_2$

$\frac{dx2}{dt} = e^t x_1 + 3x_2$

(e) $\frac{dx_1}{dt} = - x_2$

$\frac{dx_2}{dt} = (e^t + 3t)x_2$

5. The critical points of the system $x' = x - xy$ are

$y' = y\,(x+2)$

(a) only (0,0)

(b) (0,0) and (1,2)

(c) only (–2,1)

(d) (0,0) and (–2,1)

(e) (1,2) and (–2,1)

6. A fundamental matrix $\Psi(t)$ for the system $X' = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} X$ is

(a) $\begin{pmatrix} e^{-t} & e^{2t} \\ 0 & 3e^{2t} \end{pmatrix}$ (b) $\begin{pmatrix} e^{2t} & e^{-t} \\ 0 & 3e^{-t} \end{pmatrix}$ (c) $\begin{pmatrix} e^{2t} & e^{-t} \\ 0 & -e^{-t} \end{pmatrix}$

(d) $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{pmatrix}$ (e) $\begin{pmatrix} e^{-t} & e^{2t} \\ 0 & e^{2t} \end{pmatrix}$

7. The matrix $A = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$ has eigenvalues $r = -1 \pm 2\,i$.

A real solution of the system $X' = AX$ is

(a) $e^{-t} \begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix}$ (b) $e^{-t} \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$ (c) $e^{-t} \begin{pmatrix} 2\cos 2t \\ \sin 2t \end{pmatrix}$

(d) $e^{-2t} \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$ (e) $e^{-2t} \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$

8. One solution of the system $X' = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ is $X^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$. A second solution $X^{(2)}$ is

(a) $\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$ (b) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} t\, e^t$

(c) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + \begin{pmatrix} 1 \\ 1 \end{pmatrix} t\, e^t$ (d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} t\, e^t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^t$

(e) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} t\, e^t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^t$

II Partial Credit

9. Given the almost linear system

$$x' = (y - 1)\cos x$$

$$y' = \sin x + y - 1$$

(a) (2 points) Verify that x=0, y=1 is a critical point.

(b) (6 points) Find the linear system corresponding to the given system near (0,1).

(c) (6 points) Determine the type and stability of the critical point (0,1).

10. Two linearly independent solutions of the system

$$X' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} X \qquad \text{are}$$

$$X^{(1)} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} \quad \text{and} \quad X^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}.$$

(a) (5 points) A fundamental matrix Ψ for this system such that $X(t) = \Psi(t)\,C$ is

$\Psi(t) =$ ______________________________

(b) (10 points) Setting $X_p = \Psi(t)\,U$ and using the method of variation of parameters for the non-homogeneous system

$$X' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 0 \\ e^{2t} \end{pmatrix},$$

find the equation satisfied by the components u'_1 u_2', of U'.

(c) (7 points) Using the results of part (b) find a particular solution X_p for the non-homogeneous system.