Multiple Choice 8 points each.

1. The Laplace transform of $f(t) = t^3 e^{2t}$ is

(a) $\frac{6}{(s-2)^3}$ (b) $\frac{3}{(s-2)^4}$ (c) $\frac{6}{(s-2)^4}$ (d) $\frac{6}{(s+2)^4}$ (e) $\frac{1}{(s-2)^4}$

2. If f (t) has Laplace transform $F(s) = \frac{s}{s^3+1}$ then the Laplace transform of $e^{2t}f(t)$ is

(a) $\frac{s-2}{(s-2)^3+1}$ (b) $\frac{s}{s^3+2}$ (c) $\frac{s}{(s-2)^3+1}$

(d) $\frac{s+2}{(s+2)^3+1}$ (e) $\frac{2s}{4s^3+1}$

3. The inverse Laplace transform of $F(s) = \frac{2e^{-2s}}{s^2-4}$ is

(a) $u_2(t)\cosh(2t-2)$ (b) $u_2(t)\sin(2t-2)$ (c) $u_2(t)\sinh(2t+2)$
(d) $u_2(t)\cosh 2(t-2)$ (e) $u_2(t)\sinh 2(t-2)$

4. The value of the improper integral

$$\int_0^{\infty} e^{(t-2)^2} \sin\frac{\pi t}{6}\, \delta(t-3)\, dt \text{ is}$$

(a) 0 (b) e (c) $\frac{e^2}{6}$ (d) ∞ (e) $e^{(t-2)^2}\sin\frac{\pi t}{6}$

5. The solution of the initial value problem
$y'' + y = \delta(t-\pi)\cos t$; $y(0) = 0$, $y'(0) = 1$ is

(a) $\sin t + u_\pi(t)\cos t$ (b) $2\sin t$ (c) $2\cos t$
(d) $\sin t + u_\pi(t)\sin t$ (e) $\cos t + u_\pi(t)\cos t$

6. The Laplace transform of $f(t) = \int_0^t e^{-(t-\tau)} \sin\tau \, d\tau$

(a) $\dfrac{1}{(s+1)(s^2+1)}$ (b) $\dfrac{1}{s(s^2+1)}$ (c) $\int_0^t e^{-s} \sin s \, ds$

(d) $\dfrac{1}{(s-1)(s^2+1)}$ (e) $\dfrac{s}{(s+1)(s^2+1)}$

7. The largest linearly independent subset of the set of vectors
$X^{(1)} = (1,1,1)$, $X^{(2)} = (5,-2,5)$, $X^{(3)} = (1,-1,1)$ $X^{(4)} = (1,0,1)$
has exactly

(a) no members (b) one member (c) three members
(d) four members (e) two members

8. One solution of $X' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} X$ is
$X^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$. A linearly independent second solution $X^{(2)}$ is

(a) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-3t} - \begin{pmatrix} 0 \\ \frac{1}{4} \end{pmatrix} e^{-3t}$ (b) $\begin{pmatrix} 4 \\ 1 \end{pmatrix} te^{-3t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$

(c) $\begin{pmatrix} 1 \\ \frac{3}{4} \end{pmatrix} te^{-3t} - \begin{pmatrix} 0 \\ \frac{1}{4} \end{pmatrix} e^{-3t}$ (d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-3t} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} e^{-3t}$

(e) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-3t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$

9. A real solution of $X' = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix} X$ is

(a) $\begin{pmatrix} 2\cos 2t \\ -\sin 2t \end{pmatrix} e^{-t}$ (b) $\begin{pmatrix} 2\cos 2t \\ \sin 2t \end{pmatrix} e^{-t}$

(c) $\begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix} e^{-t}$ (d) $\begin{pmatrix} \sin 2t \\ 2\cos 2t \end{pmatrix} e^{t}$ (e) $\begin{pmatrix} -2\sin 2t \\ \cos 2t \end{pmatrix} e^{-t}$

10. A fundamental system of solutions of $X' = \begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix} X$ is $X^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t}$, $X^{(2)} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} e^{-3t}$. A fundamental matrix $\Phi(t)$ such that $\Phi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is

(a) $\begin{pmatrix} e^{-3t} & 0 \\ 0 & e^{2t} \end{pmatrix}$ (b) $\begin{pmatrix} e^{2t} & 5(e^{2t} - e^{-3t}) \\ 0 & e^{-3t} \end{pmatrix}$ (c) $\begin{pmatrix} (2e^{2t} - e^{-3t}) & 0 \\ (e^{-3t} - e^{2t}) & e^{-3t} \end{pmatrix}$

(d) $\begin{pmatrix} e^{2t} & \frac{1}{5}(e^{2t} - e^{-3t}) \\ 0 & e^{-3t} \end{pmatrix}$ (e) $\begin{pmatrix} e^{2t} & (e^{2t} - 1) \\ (e^{-3t} - e^{2t}) & e^{-3t} \end{pmatrix}$

$$\begin{pmatrix} e^{-3t} & 0 \\ 0 & e^{2t} \end{pmatrix}$$

11. The point (–3,7) is a critical point for the nonlinear system

$$x' = x^2 + 5x + 6$$
$$y' = x + y - 4$$

This point is

(a) A stable spiral point (b) A saddle point
(c) An unstable improper node (d) a center
(e) A stable proper node

12. Suppose that $\phi(x)$ satisfies the initial value problem

$$y' = y + 4t \qquad y(0) = 3$$

Use Euler's method with stepsize h = 1 and two iterations to estimate $\phi(2)$.
The result is $\phi(2) \cong$

(a) 2 (b) 4 (c) 0 (d) –7 (e) 16

13. The value of the coefficient b_7 in the sine series with period 2π

$\sum_{n=1}^{\infty} b_n \sin nx$ for $f(x) = 3\pi$ on the interval $0 \le x < \pi$ is

(a) $\frac{12}{7}$ (b) $\frac{4}{7}$ (c) $\frac{-4}{7}$ (d) 0 (e) $\frac{3\pi}{7}$

14. Let $f(x) = \begin{cases} 5\cos\frac{x}{3} & 0 \le x < \frac{3\pi}{2} \\ 0 & \frac{3\pi}{2} \le x < 3\pi \end{cases}$

be periodic with period 3π. The value of $f(x)$ at $x = 3\pi$ so that the Fourier series for f converges to f at every point is $f(3\pi) =$

(a) $\frac{5}{2}$ (b) 5 (c) 0 (d) $\frac{-5}{2}$ (e) -5

15. For the heat conduction problem with insulated ends

$u_{xx} = u_t$, $u_x(o,t) = 0$, $u_x(3,t) = 0$

the solutions are of the form (set $\lambda = \frac{n\pi}{3}$) : $u_n(x,t) =$

(a) $e^{-\lambda^2 t} \sin 3\lambda x$ (b) $e^{-\lambda^2 t} \cos \lambda x$ (c) $e^{-3\lambda^2 t} \cos \lambda t$
(d) $e^{-\lambda^2 t^2} \sin \lambda x$ (e) $e^{-\lambda^2 x^2} \cos \lambda t$

Partial Credit Problems

16. A 75 cm bar of metal for which $\alpha^2 = 2$ has one end held at 5ºc and the other held at 40ºc. The bar is initially heated in such a way that

$$u(x,0) = f(x) = -2 \sin\left(\frac{6\pi x}{75}\right) .$$

(a) (18 points) Solve the heat conduction problem for u (x,t)

(b) (6 points) find the steady state temperature distribution in the bar

17. Let $f(x) = 1 - x^2$ for $0 \le x \le 2$.

(6 points) Define a periodic extension of f with period 4 which is neither even nor odd. Sketch the graph for three periods; one of which is to the left of x = 0 and one is to the right.