MATH 325
September 30, 1994

TEST I

1. (9 points) Find the general solution of the equation

$$y''' - 3y'' + 4y' - 2y = 0 .$$

a. $C_1 e^x + C_2 x e^x + C_3 x^2 e^x$

b. $C_1 e^x + C_2 e^x \cos x + C_3 e^x \sin x$

c. $C_1 + C_2 \cos x + C_3 \sin x$

d. $C_1 e^{-x} + C_2 e^{-x} \cos x + C_3 e^{-x} \sin x$

e. $C_1 x \cos x + C_2 x \sin x + C_3 e^{-x}$

2. (9points) Determine a suitable form for a particular solution of

$$y'' - y' = x\,e^x$$

using the method of undetermined coefficients.

a. $Ax^2\,e^x + Bxe^x$

b. $A\,e^x + Bxe^x$

c. $A\,x + Be^x$

d. $A + Be^x$

e. $A + Bxe^x$

3. (9points) Let f(t) be the function defined by

$$f(t) = \begin{cases} 1, & t < 1 \\ 2, & t \geq 1 \end{cases} .$$

From the definition of the Laplace transform, or otherwise, find the Laplace transform F(s) of f(t).

a. $\frac{e^{-s} + 1}{s}$

b. $\frac{e^{s} - 1}{s}$

c. $\frac{1 + e^{s}}{s}$

d. $\frac{e^{s} - e^{-s}}{s}$

e. $\frac{e^{s} + e^{-s}}{s}$

4. (9 points) Find the inverse Laplace transform of the function

$$F(s) = \frac{1}{s^2 - 2s + 5} + 2\frac{e^{-(s+1)}}{s + 1}$$

a. $\frac{1}{2} e^{t} \sin 2t + 2 e^{-t} u_1(t)$

b. $e^{t} \cos 2t + e^{-t} u_1(t)$

c. $\frac{1}{2} e^{-t} \sin 2t + u_1(t) (t - 1)$

d. $\frac{1}{2} e^{t} \sinh t + e^{-t} u_1 (t)$

e. $e^{t} \cosh t - u_1(t) (t - 1)$

5. (9 points) Using the definition, compute the Laplace transform of

$$f(t) = \delta(t - 1) u_{\frac{1}{2}}(t) e^{t-1} .$$

a. e^{-s}

b. $e^{-(s + 1)}$

c. $u_{\frac{1}{2}}(s) + 1$

d. $\delta(s)$

e. 0

6. (9 points) The following form of the general solution of the non-homogeneous equation $y'' + y' - 6y = x$ is found using the method of undetermined coefficients. Which one is it?

a. $C_1 e^{-3x} + C_2 e^{2x} - \frac{1}{6}\left(x + \frac{1}{6}\right)$

b. $C_1 e^{-3x} + C_2 e^{2x} + x + \frac{1}{36}$

c. $C_1 \left(e^{-3x} + e^{2x}\right) - C_2 x + \frac{1}{6}$

d. $C_1 e^{2x} + C_2 x e^{2x} + e^{-3x}$

e. $C_1 e^{-3x} + C_2 x e^{-3x} + e^{2x}$

7. (9 points) Find the solution of the intial value problem

$$y'' + 3y = \cos t$$
$$y(0) = 0, y'(0) = 0 \quad .$$

a. $\frac{1}{2} \cos t - \frac{1}{2} \cos \sqrt{3}\ t$

b. $1 - \cos 2t$

c. $\sin \sqrt{3}\ t + \cos t$

d. $\frac{1}{2} \sin t + \frac{1}{4} \sin \sqrt{3}\ t$

e. $\cos \sqrt{3}\ t$

8. (9 points) Find the Wronskian determinant of the three functions

$$\{\cos^2 x, \sin^2 x , 1 \} .$$

a. 0

b. tan x

c. sec x

d. cos x sin x

e. 1

PART B (PARTIAL CREDIT)

9. (14 points) Use the method of variation of parameters to find a particular solution of the equation $y''' - y' = 1$.

CAUTION: No credit for any other method.

10. (14 points) Find the solution of the initial value problem

$$y'' - 4y = \delta(t-1)$$

$$y(0) = 0, y'(0) = 0 \quad .$$