MATH 325

1. (9 points) Compute the Wronskian of the vectors

$$\mathbf{x}^{1} = \begin{bmatrix} t^2 \\ t \\ 0 \end{bmatrix} \qquad \mathbf{x}^{(2)} = \begin{bmatrix} -t \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} 0 \\ 0 \\ e^t \end{bmatrix} .$$

a. $2t^2e^t$ b. 0 c. $(t^2 + 1)\, e^t$ d. $(t^3 - t^2)\, e^t$ e. $-2t\, e^t$

2. (9 points) Find all of the eigenvalues of the matrix $\begin{bmatrix} -1 & -3 & 2 \\ 0 & 2 & 0 \\ 3 & 0 & -2 \end{bmatrix}$ .

a. 1, 2 and – 4 b. 1, 2 and 3 c. 1, 2 and –3

d. –1, –2 and 4 e. 1, 2 and 4

3. (9 points) The matrix $A = \begin{bmatrix} -2 & 1 & 4 \\ 0 & -2 & -3 \\ 0 & 0 & 2 \end{bmatrix}$ has $r = -2$ as a repeated

eigenvalue of multiplicity two. Find all of the eigenvectors v of A corresponding to this eigenvalue (the arbitary constants appearing in each of the answers may be assumed to be not all zero).

a. $v = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

b. $v = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

c. $v = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

d. $v = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

e. $v = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

4.(9 points) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$ .

a. $\begin{bmatrix} \frac{4}{10} & \frac{-2}{10} \\ \frac{3}{10} & \frac{1}{10} \end{bmatrix}$

b. $\begin{bmatrix} \frac{1}{10} & \frac{-2}{10} \\ \frac{3}{10} & \frac{4}{10} \end{bmatrix}$

c. $\begin{bmatrix} \frac{-1}{10} & \frac{-2}{10} \\ \frac{3}{10} & \frac{-4}{10} \end{bmatrix}$

d. $\begin{bmatrix} \frac{4}{10} & \frac{-2}{10} \\ \frac{-3}{10} & \frac{1}{10} \end{bmatrix}$

e. $\begin{bmatrix} \frac{3}{10} & \frac{-4}{10} \\ \frac{1}{10} & \frac{-2}{10} \end{bmatrix}$

5.(9 points) Find the general solution of $x' = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} x$ .

**NOTE:** Be aware that the general solution that you find should be one of the answers below. . . . perhaps after a suitable re-naming of your arbitrary constants.

a. $e^{t} \begin{bmatrix} -c_1 \sin 2t + c_2 \cos 2t \\ c_1 \cos 2t + c_2 \sin 2t \end{bmatrix}$

b. $e^{-t} \begin{bmatrix} -c_1 \sin 2t + c_2 \cos 2t \\ c_1 \cos 2t + c_2 \sin 2t \end{bmatrix}$

c. $e^{t}\begin{bmatrix} c_1 \sin 2t + c_2 \cos 2t \\ c_1 \cos 2t + c_2 \sin 2t \end{bmatrix}$

d. $e^{2t}\begin{bmatrix} -c_1 \sin t + c_2 \cos t \\ c_1 \cos t + c_2 \sin t \end{bmatrix}$

e. $e^{t}\begin{bmatrix} c_1 \sin t + c_2 \cos t \\ -c_1 \cos t + c_2 \sin t \end{bmatrix}$

6. (9 points) Which statement is true for the following system

$$x' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} x \quad .$$

a. The origin is a saddle point

b. The origin is an improper node

c. The origin is a proper node

d. The origin is a spiral point

e. The origin is not a critical point

7. (9 points) Using row reduction on the augmented matrix (or otherwise) find the number k such that the system

$$x_1 - 2x_2 + 3x_3 = k$$

$$-x_1 + x_2 - 2x_3 = -1$$

$$2x_1 - x_2 + 3x_3 = 0$$

has a solution.

a. k= 3 b. k= 2 c. k= 1 d. k= 0 e. k= –1

8.(9 points) The matrix $A = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix}$ has $r = -3$ as an eigenvalue of multiplicity two with corresponding eigenvector $\xi = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Which of the pictures below most closely resembles the phase portrait of the system

$$\mathbf{x}' = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix} \mathbf{x} .$$

a.

b.

c.

d.

e.

PARTIAL CREDIT

9. (14 points) Find the general solution of the system $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} \mathbf{x}$ .

10.(14 points) Find the general solution of the non-homogeneous system

$$x' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 2e^{-2t} \\ 0 \end{bmatrix}$$

evaluating all integrals that occur.

Hint: A fundamental matrix of the system $x' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x$

is $\psi(t) = \begin{bmatrix} e^{-3t} & e^{-t} \\ -e^{-3t} & e^{-t} \end{bmatrix}$ .