MATH 325 TEST 3
December 7, 1994

1. (9 points) For the system

$$x' = x - y$$
$$y' = -x + 3x^2 y$$

the critical point (0, 0) is :

a. a proper node b. a saddle point c. an improper node

d. a spiral point e. a centre

2. (9 points) List the set of <u>all</u> critical points of the system

$$x' = \sin^2 y$$
$$y' = (x^2 - 4)(y^2 - 1) .$$

a. (2, 1), (2, −1), (−2, 1) and (−2, − 1)

b. $(2, \pm n\pi)$ and $(-2, \pm n\pi)$

c. (2, 0) and (−2, 0)
$\left(2, \pm n\frac{\pi}{2} \right)$

d. $\left(-2, \pm n\frac{\pi}{2} \right)$ and

e. $(2, \pm n\pi)$ and (− 2, 0)

Remark: In these answers n stands for all positive integers and zero.

3. (9points) Which of the sketches below best describes the trajectories of the system

$$x' = x - 10y + \sin^2 y$$

$$y' = 10x + y + \sin^2 x$$

in the vicinity of the critical point (0, 0) ?

a.

b.

c.

d.

e.

4. (9 points) Let u (x, t) denote the solution of the heat conduction problem

$$u_{xx} = u_t \qquad 0 < x < 1,\ t > 0$$

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = \sin \pi x - \sin 2\pi x \ .$$

Then the value of u (x, t) when $x = \frac{1}{2}$ is :

a. 0 b. $e^{-4\pi^2 t}$ c. $e^{4\pi^2 t}$ d. $e^{-\pi^2 t}$ e. $-e^{-\pi^2 t}$

5. (9 points) Which of the following is true for the function

$$f(x) = x^2 + \cos x + \sin^2 x.$$

a. It has period 2π b. It is neither even nor odd

c. It has a periodic 1 d. It is even

e. It is odd

6. (9 points) Let f and g be odd functions with the same period.

a. The Fourier series of f –g contains only nonzero cosine terms

b. The Fourier series of f g contains both nonzero cosine and nonzero sine terms

c. The Fourier series of f +g contains only nonzero sine terms

d. The Fourier series of f –g contains both nonzero cosine and nonzero sine terms

e. The Fourier series of f g contains only nonzero sine terms

7. (9 points) The value of the intergral $\int_{-4}^{4} \cos 17\,\pi\, x \sin 17\,\pi\, x\, dx$

a. 8 b. 0 c. $\frac{1}{8}$ d. $\frac{17}{4}$ e. $\frac{4}{17}$

8. (9 points) Find the coefficient b_3 of the Fourier series of the function f (x) defined by

$$f(x) = x \qquad -1 \le x < 1$$

$$f(x+2) = f(x) \qquad \text{for all } x\,.$$

a. $\frac{1}{3\pi}$ b. $\frac{-2}{3\,\pi}$ c. 0 d. $\frac{2}{3\pi}$ e. $-\frac{1}{3\,\pi}$

PARTIAL CREDIT

9. (14 points) The point (0, 0) is a critical point of the system

$$x' = - x^5$$

$$y' = - y^5$$

Verify that this is an asymptotically stable critical point of this system by choosing a suitable Liapunov function of the form $V(x, y) = a x^2 + b y^2$.

10. (14 points) Write down the solution (as an infinite series) of the following heat conduction problem:

$$u_{xx} = u_t \quad \text{for} \quad 0 < x < 1 , \quad t > 0 .$$

$$u(o, t) = u(1, t) = 0 \quad \text{for } t > 0$$

$$u(x, 0) = 1 \quad \text{for} \quad 0 < x < 1 .$$

Evaluate all integrals arising.