MATH 325 (5th try 12-13)
FINAL

1. The characteristic equation of a certain fourth order linear homogeneneous differential equation is $(\lambda^2 + 4)(\lambda + 2)^2 = 0$

Find the fundamental solution

a. $c_1 e^{-2x} + c_2 xe^{-2x} + c_3 \cos 2x + c_4 \sin 2x$
b. $c_1 e^{-2x} + c_2 e^{2x} + c_3 \cos 2x + c_4 \sin 2x$
c. $c_1 xe^{-2x} + c_2 x^2 e^{-2x} + c_3 \cos 2x + c_4 \sin 2x$
d. $c_1 + c_2 e^{-2x} + c_3 \cos 2x + c_4 \sin 2x$
e. $c_1 e^{-2x} + c_2 x e^{-2x} + c_3 x \cos 2x + c_4 x \sin 2x$

2. Determine a suitable form for a particular solution of $y'' + y' = xe^{-x}$ using the method of undetermined cofficients.

a. $a x^2 e^{-x} + bxe^{-x}$

b. $a xe^{-x} + be^{-x}$

c. $a x + be^{-x}$

d. $a + be^{-x}$

e. $a + bxe^{-x}$

3. Find the solution of the initial value problem

$$y'' + 4y = \sin t$$
$$y(0) = 0 \qquad y'(0) = 0$$

a. $-\frac{1}{6}\sin 2t + \frac{1}{3}\sin t$

b. $1 - \cos 2t$

c. $\frac{1}{2}\sin 2t - \frac{1}{3}\sin t$

d. $\frac{1}{6}\sin 2t - \frac{1}{3}\sin t$

e. $\frac{1}{6}\sin 2t + \frac{1}{3}\sin t$

4. Find the inverse Laplace transform of $F(s) = \dfrac{e^{-\pi s}}{s^2 + 2s + 2}$

a. $u_\pi(t)\, e^{-(t-\pi)} \sin(t - \pi)$

b. $u_\pi(t)\, e^{-(t-\pi)} \cos(t - \pi)$

c. $u_\pi(t)\, e^{-t} \sin t$

d. $u_\pi(t)\, e^{-t} \cos t$

e. $u_\pi(t)\, e^{t-\pi} \sin(t - \pi)$

5. Find the solution of the initial value problem $y'' + y = \delta(t - \pi)$, $y(0) = 0,\ y'(0) = 0$.

a. $u_\pi(t) \sin(t - \pi)$

b. $u_\pi(t) \cos(t - \pi)$

c. $u_\pi(t)\, e^{-\pi} \sin(t - \pi)$

d. $u_\pi(t)\, e^{-(t - \pi)} \sin(t - \pi)$

e. $\sin(t - \pi)$

6. Find all eigenvalues of the matrix

$$\begin{pmatrix} 1 & 4 & -2 \\ 0 & 1 & 0 \\ -3 & 0 & 2 \end{pmatrix} .$$

a. 1, −1, 4

b. 1, 1, 4

c. 1, −1, −4

d. −1, −1, 4

e. −1, −1, −4

7. Find the inverse of $\begin{pmatrix} 1 & -1 & -1 \\ 3 & -2 & -2 \\ 0 & 2 & 3 \end{pmatrix}$

a. $\begin{pmatrix} -2 & 1 & 0 \\ -9 & 3 & -1 \\ 6 & -2 & 1 \end{pmatrix}$ b. $\begin{pmatrix} 2 & 1 & 0 \\ 3 & 1 & -1 \\ 4 & -2 & 1 \end{pmatrix}$ c. $\begin{pmatrix} 2 & -1 & 0 \\ 9 & -3 & 1 \\ 6 & -2 & 1 \end{pmatrix}$

d. $\begin{pmatrix} 2 & -1 & 0 \\ 9 & -3 & 1 \\ -6 & 2 & -1 \end{pmatrix}$ e. $\begin{pmatrix} 1 & 0 & 1 \\ 3 & 4 & 2 \\ 1 & -1 & 1 \end{pmatrix}$

8. Use row reduction on the system $\begin{cases} x_1 - 2x_2 + 3x_3 = 3 \\ -x_1 + x_2 - 2x_3 = 1 \\ 2x_1 - x_2 + 3x_3 = 0 \end{cases}$

to pick the right answer.

a. $\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ b. $\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ c. $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

d. $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ e. The system has no solution.

9. Find the general solution of $x' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x$

a. $c_1 \begin{pmatrix} 2\cos t - \sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \cos t + 2\sin t \\ \sin t \end{pmatrix}$

b. $c_1 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2 \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$

c. $c_1 \begin{pmatrix} -\sin t \\ \cos t + \sin t \end{pmatrix} + c_2 \begin{pmatrix} \cos t \\ \cos t - \sin t \end{pmatrix})$

d. $c_1 \begin{pmatrix} e^t \\ 2e^t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t + 2\sin t \end{pmatrix}$

e. $c_1 \begin{pmatrix} \cos t \\ 2\cos t - \sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t + 2\sin t \end{pmatrix}$

10. Find the general solution of the system $x' = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix} x$

a. $c_1 e^{-2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} t \\ 1 \end{pmatrix}$

b. $c_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} t \\ 1 \end{pmatrix}$

c. $c_1 e^{-2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} t \\ 1 \end{pmatrix}$

d. $c_1 e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ t \end{pmatrix}$

e. $c_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} t \\ 1 \end{pmatrix}$

11. $\left\{ \begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix}, \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix} \right\}$ is a fundamental system of solutions for $x' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} x$

Let $x = u_1 \begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix} + u_2 \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix}$, as in the method of varation of parameters,

be a solution to $x' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}$.

Find u_1'

a. $\frac{1}{2} e^{2t}$ b. e^{2t} c. $\frac{1}{2} e^{-2t}$ d. $\frac{1}{3} e^{2t}$ e. $\frac{1}{4} e^{2t}$

12. Consider the problem $\begin{cases} y' = -t + \sin y \\ y(0) = 0 \end{cases}$

Using the Euler method with step size h = 1, an approximation of y_2 of the solution at t = 2 is

a. -1 b. $\sin 1$ c. $1 - \sin 2$ d. 1 e. -2

13. What is the type of the critical point of $x' = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix} x$?

a. improper node b. saddle point c. spiral point

d. center e. proper node

14. What does the Liapunov function $V(x,y) = x^2 + y^2$ tell us about the critical point (0, 0) of the system

$$\begin{cases} x' = -x^3 + 2y^3 \\ y' = -2xy^2 - 2y^3 \end{cases}$$

a. it is asymptotically stable b. it is a center c. it is unstable

d. it is stable, but not asymptotically stable e. it is a saddle point

15. Which of the following is not periodic?

a. $\cos x + \sin \pi x$ b. $2 + \sin \pi x$ c. $\frac{1}{2} \cos x - \sin^5 x$

d. $e^{\sin x} \cos^2 x$ e. $|\cos x|$

16. Consider the function defined by $f(x) = \begin{cases} x & \text{if } -1 \le x \le 0 \\ 3 & \text{if } 0 < x \le 1 \end{cases}$ and $f(x+2) = f(x)$, for all x.

To what value does the Fourier series of $f(x)$ converge at $x = 0$?

a. $\frac{3}{2}$ b. $\frac{1}{2}$ c. 1 d. 0 e. 2

17. Find the value of the coefficient a_3 in the Fourier series expansion of the even function with period 2 which coincides with $f(x) = 3$ for $0 < x < 1$.

a. 0 b. 3 c. − 1 d. $\frac{3}{2}$ e. 1

18. What is the Fourier coefficient a_o of the periodic function

$$\begin{cases} f(x) = 1 + \sin^7 x, & -1 \le x < 1 \\ f(x+2) = f(x) \text{ for all } x. \end{cases}$$

a. 2 b. $\pi + 2$ c. $1 + \frac{\pi}{2}$ d. 0 e. 1

19. A metal bar of unit length with x $\alpha = 1$ is heated with one end held at 10°C and the other end held at 20°C. Suppose the initial temperature is

$$u(x, 0) = \sin \pi x + 10x + 10.$$

What is $u\left(\frac{1}{2}, 1\right)$?

a. $e^{-\pi^2} + 15$

b. $e^{-\pi^2}$

c. 0

d. $20 + e^{-\pi^2}$

e. $20 - e^{-\pi^2}$

20. What is the 2x2 matrix of the corresponding linear system of

$$\begin{cases} x' = x + x^4 \\ y' = 2y + xy^3 \end{cases}$$

at the critical point $(0, 0)$?

a. $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

c. $\begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}$

d. $\begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$

e. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$